

DECEMBER 1977

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TR 7-77

ACN 22273

# OVERVIEW OF ATMT AND ANALYSIS OF SUBPHASE IIB

TECHNICAL REPORT TR 7-77

UNITED STATES ARMY  
COMBINED ARMS CENTER

COMBINED ARMS  
COMBAT DEVELOPMENTS ACTIVITY

COMBAT OPERATIONS ANALYSIS DIRECTORATE

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER Technical Report TR 7-77 ✓	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Overview of ATMT and Analysis of Subphase IIB.		3. TYPE OF REPORT & PERIOD COVERED Final
7. AUTHOR(s) Michael R. Anderson and Robert J. Lenz		6. PERFORMING ORG. REPORT NUMBER
9. PERFORMING ORGANIZATION NAME AND ADDRESS US Army Combined Arms Combat Developments Activity Combat Operations Analysis Directorate ATTN: ATCA-CAT-D, Fort Leavenworth, KS 66027		8. CONTRACT OR GRANT NUMBER(s)
11. CONTROLLING OFFICE NAME AND ADDRESS Same as above		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS ACN 22273
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		12. REPORT DATE December 1977
		13. NUMBER OF PAGES 106
		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Acceleration, angular acceleration, angular velocity, antitank, antitank guided missile, cubic spline, DRAGON, gunner error, gunner tracking error, infrared beacon, intervisibility combinations, line of sight (LOS), maneuver, missile error, M60A1 tank, Range Measuring System, Shillelagh, TOW, velocity, XM800 Scout, XM808 TWISTER		
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Results of the analysis of the data collected in Subphase IIB are provided in three areas: line-of-sight pairings with firing sequences, target vehicle velocity and acceleration summaries, and relationships among maximum gunner error and target vehicle motion.

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6) OVERVIEW OF ATMT AND ANALYSIS OF SUBPHASE B

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ACN 22273

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## FOREWORD

This analysis was conducted by the Test Plans and Data Management Branch of the Combat Operations Analysis Directorate, US Army Combined Arms Combat Developments Activity, in support of the Department of the Army directed Antitank Missile Test (ATMT). Authority for this experiment and analysis is contained in the Outline Test Plan, Antitank Missile Test (ATMT), FC 019, 29 January 1975. This technical report was completed in December 1977.

The authors wish to express their appreciation to LTC Merrill Steele, MAJ Joseph Terry, Dr. David Bash, Dr. Edward Inselmann, Mr. Jack Low, and Mr. William Martin for their direct involvement in this report and the technical expertise they provided throughout the study. Thanks are also extended to Mr. James Brown and the entire staff of the Systems Simulation and Support Branch, USAMSAA, Aberdeen Proving Ground, Maryland whose insights contributed to the completion of this analysis, and to Mrs. Rosalie Fulks, whose talents transferred illegible manuscript into typed copy.

## ABSTRACT

This report contains an overview of all phases and subphases of the Antitank Missile Test (ATMT) with a detailed discussion of the analysis of the data collected during Subphase IIB. The purpose of ATMT was to determine the degradation of the TOW, DRAGON, and Shillelagh missile systems caused by target vehicle evasive maneuvers. The target vehicles used in all subphases were the M60A1 tank, XM800 Scout, and the XM808 TWISTER. The ATMT methodology includes the use of hybrid missile simulations, actual gunner tracking error, vehicle evasive maneuvers in an operational environment, and data collection from field testing. Thus, the field test provided realistic data on all but the live firing of the missiles. The verified missile simulations, using field test data as input, were then substituted for actual live firing.

Results of the analysis of the data collected in Subphase IIB are provided in three areas: line-of-sight pairings with firing sequences, target vehicle velocity and acceleration summaries, and relationships among maximum gunner error and target vehicle motion.

## EXECUTIVE SUMMARY

1. BACKGROUND. The Antitank Missile Test (ATMT) was a two-phased effort to consider the two basic components of an antitank guided missile (ATGM) system, the launcher-man system (or gunner input) and the missile-tracker system (or missile input). The missile systems considered in ATMT were the TOW, DRAGON, and Shillelagh. Phase I, completed in December 1974, identified an array of candidate maneuvers to be used in the Phase II field test. These maneuvers were ranked by their ability, assuming perfect gunner tracking, to tax missile response capabilities to the greatest degree. The final maneuvers selected for the field test were variations of the serpentine, swerve, and fast turn.

2. PURPOSE. The Phase II field test and analysis was designed to determine the degradation of the TOW, DRAGON, and Shillelagh ATGM system capabilities as a result of target maneuver.

### 3. OBJECTIVES.

a. Objective 1. Obtain data to determine if ATGM systems can track targets that are executing maneuvers thought to be limiting to the technical capabilities of the TOW, DRAGON, and Shillelagh systems.

b. Objective 2. Obtain aiming error data to determine if tanks with conventional weapons systems can engage the same type of maneuvering targets.

c. Objective 3. Provide data that may be used to:

(1) Develop product improvements for existing antitank conventional and guided missile systems.

(2) Develop follow-on requirements for antitank systems.

(3) Improve antitank and armor crew programs for training.

(4) Assist in improving existing combat simulation models.

(5) Evaluate effects of target vehicle mobility on gunner tracking performance.

### 4. SCOPE OF EXPERIMENT.

a. The Phase II analysis was conducted by subphase. This report contains the summary of the analysis for Subphase IIB and is unclassified. The summary of the analysis of the remaining subphases is being published by AMSAA as a separate volume and is classified SECRET.

b. Each missile system under consideration contains an infrared (IR) source on the missile. The function of the IR source is to allow the missile tracker to sense any discrepancy between the missile flight path

and the line of sight established by the gunner. If a discrepancy is detected, a corrective command is automatically transmitted to the missile. In lieu of firing live missiles, the ATMT methodology measured gunner error via the corrective command generated by the missile tracker. To capture gunner error instead of corrections to the ATGM, an IR beacon was mounted on top of the target vehicles so that it was visible from all aspects. The error signal generated was the difference between the tracker-beacon line and the gunner's aim line. With the bias for beacon mounting position removed, this error signal became the gunner tracking error from the ideal aim point.

c. Subphase IIA was designed to evaluate gunner tracking when target vehicles performed exact imitations of the mathematically generated maneuvers selected for field testing in Phase I. Each path was visually marked for drivers of the target vehicles. Vehicle paths consisted of from two to four maneuvers connected by 50-meter straight stabilization segments. At no time was line of sight broken between the weapon systems and the target vehicles.

d. Subphase IIB was the most operationally oriented approach to evaluating evasive maneuvers. Drivers of the target vehicles were in a free-play mode, which allowed them to utilize terrain and cover to break line of sight while traversing the required terrain. Gunners indicated times at which they felt weapons should be fired.

e. Subphase IIC employed the three maneuvers from Subphase IIA, the swerve, fast turn, and random serpentine. As in Subphase IIA, continuous line of sight was maintained and 50-meter straight stabilization segments between maneuvers were used. However, drivers were told the sequence of maneuvers to execute and the avenue of approach to take. They then performed their interpretation of the required maneuvers.

f. The target vehicles used in all three subphases were the M60A1 tank, XM800 Scout, and the XM808 TWISTER.

## 5. SUBPHASE IIB ANALYSIS.

a. Ideally, the analysis of the IIB data would have consisted of missile simulations using the actual gunner tracking record, the gunner firing times, and the target vehicle's recorded path. However, because of the line-of-sight interruptions, significant difficulties were encountered early in the effort to digitize the analog gunner error data from this subphase. In addition, this effort would have included data gaps and AMSAA's antitank missile simulations could not have been used during breaks in LOS. Therefore, the decision was made not to digitize the analog data.

b. Since the missile simulations could not be performed, the IIB analysis was revised. This revision focused on three main efforts:

(1) A line-of-sight (LOS) analysis to determine the probability, displayed in the field experiment data, of successfully tracking the target



vehicle from the simulated firing time to the projected impact time without losing LOS.

(2) An analysis of the velocities and accelerations of the target vehicles to portray their performance in the field experiment, for potential comparison against their projected capabilities.

(3) An analysis of maneuver, velocity/acceleration, and raw tracking data to identify any observable correlations in that data.

## 6. CONCLUSIONS.

### a. Line-of-Sight Analysis.

(1) A maneuvering target vehicle that utilizes terrain and vegetation characteristics has the capability to impose an appreciable number of missile abort situations. The frequency and duration of target vehicle LOS interruptions, density of the vegetation, and range from the target to the gunner are influential parameters in producing these missile abort situations.

(2) TOW gunners were best able to perceive the probability of unobstructed line of sight at both the time of indicated firing and at the projected time of missile impact (LOS-LOS combination) in the firing and impact sequence at ranges less than 1,600 meters. As the range increased, Shillelagh gunners from both the M551 and M60A2 had the best perception of the LOS-LOS probability when firing.

(3) It appeared that the XM808 TWISTER was the target vehicle that most successfully disrupted the LOS-LOS combination of the DRAGON and TOW gunners firing and impact sequences. The Shillelagh gunners, from both the M551 and M60A2, had more difficulty achieving the LOS-LOS combination against the XM800 Scout than against the other two target vehicles.

### b. Target Velocities and Accelerations.

(1) The perspective and range from which a gunner observes a maneuvering target significantly affects the amount of target motion presented to the gunner.

(2) The average vehicle velocities and accelerations demonstrated on cross-country terrain are lower than might be expected considering performance standards stated for the vehicles.

(3) The differences among mean velocities and accelerations for the different target vehicles are not as pronounced as might be expected.

### c. Relationship of Maximum Gunner Error with Target Vehicle Motion.

(1) Under operational conditions and with the analysis techniques used, target vehicle velocities and accelerations show little relationship with large gunner errors.

(2) No maneuver appears to be any more significant at causing major errors in gunner tracking than any other maneuver. In fact, maneuver alone cannot be considered as a cause for significant gunner error.

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## 1. INTRODUCTION.

### a. Background.

(1) In August 1973, interest was generated at Department of the Army in the capability of a maneuvering target to degrade the effectiveness of US antitank guided missile (ATGM) systems. This interest was a result of a limited number of live firings of inert ATGM against an evasive target in the Tactical Effectiveness Testing of Antitank Missiles (TETAM) program. The Combined Arms Combat Developments Activity (CACDA) became the Training and Doctrine Command (TRADOC) proponent of a test program; and in January 1974, a multiphased test, involving the efforts of CACDA, Army Materiel Systems Analysis Activity (AMSAA), Combat Developments Experimentation Command (CDEC), and Missile Readiness Command (MIRCOM, formerly MICOM), was approved. The test methodology integrated hybrid computer missile simulations, field test data, and a knowledge of the ATGM systems and their engagement processes while allowing detailed investigation of various ATGM engagement sequences without the firing of "live" missiles.

(2) A two-phased effort was initiated to consider the two basic components of an ATGM system: the launcher-man system (or gunner input) and the tracker-missile system (or missile input). The missile systems considered in the Antitank Missile Test (ATMT) study were the TOW, DRAGON, and Shillelagh. Phase I of ATMT was completed in December 1974 and was published as an Interim Note by AMSAA (reference 1). The Phase II analysis was conducted by subphase. This report contains the summary of the analysis for Subphase IIB and is unclassified. The summary of the analysis of the remaining subphases is being published by AMSAA as a separate volume and is classified SECRET.

### b. Phase I Summary.

(1) The purpose of the Phase I effort was to identify an array of candidate maneuvers to be used by the target vehicles in the Phase II field testing. These maneuvers, once identified, were ranked by their ability to tax missile response capabilities to the greatest degree.

(2) The missile systems considered in the study detect the missile displacement off the line of sight (LOS). From these data corrective command signals are generated to bring the missile back to the LOS. The feed forward commands proportional to the LOS rates are generated in the TOW and Shillelagh systems to provide the Coriolis lateral acceleration component. The feed forward command required by the missile systems to remain on the LOS would be the only command required if the target were traveling on a circular path of constant radius to the tracker at a constant speed, resulting in an LOS angular acceleration of zero; but when that acceleration is not zero because of target acceleration, the sensitivity of the error sensing loop of the missile tracker to LOS acceleration determines the magnitude of the missile error (reference 2).

(3) To enhance the potential survivability of a target by maneuver, the vehicle should cause a continual acceleration of the LOS. This may be

accomplished by several methods, the simplest being to use the inherent internal acceleration capability of the target vehicle. A second alternative is to arrange or vary the geometry of the course traveled by the target vehicle so that a constant speed target has apparent acceleration with respect to the missile tracker. The LOS may also be accelerated by a combination of both internal vehicle acceleration and course geometry. This combination more accurately represents the situation expected in a battle-field environment, but since the acceleration capabilities of the types of targets considered provided no significant advantage when compared to the apparent acceleration resulting from the course geometry, the majority of the maneuvers investigated considered the target velocity to be constant. Specifically, nine types of maneuvers were investigated. They were as follows:

- o START
- o STOP
- o STOP/START
- o FAST TURN
- o S TURN
- o SWERVE
- o SERPENTINE
- o RANDOM SERPENTINE
- o ZIG ZAG

Complete descriptions and diagrams of the maneuver types are included in appendix A. Each basic target maneuver was varied by range, target aspect angle, velocity, and the timing of missile firing within each maneuver.

(4) To assess the capabilities of these maneuvers to degrade missile responsiveness, AMSAA constructed analog computer simulations of the three antitank missile systems that duplicated those simulations developed by the prime contractor for each system (references 3, 4, 5, and 6). Because the ATGM systems are command-to-LOS, the miss distance for a missile in terminal flight is a combination of the deviation of the missile from the LOS (missile error) and the deviation of the LOS from the target (gunner error). To investigate the effect of the maneuvers on missile responsiveness, the gunner error was assumed to be zero. By considering only perfect gunner tracking the LOS was always coincident with the target line, and the missile error became the miss distance. The LOS angular rate history of the maneuvering target relative to the missile tracker was entered into the simulations as a forcing function to establish missile flight profiles. These flight profiles represent the missile deviations from the moving LOS as a function of time and are a measure of the

effectiveness of the maneuver in keeping the missile off the LOS and off the target at projected impact. From the collected data the candidate maneuvers were then ranked by their effectiveness, and those being most effective were selected for use in the field trials (figure 1).

c. Phase II Purpose and Objectives.

(1) Purpose. Determine the degradation of the TOW, DRAGON, and Shillelagh ATGM system capabilities as a result of target maneuver.

(2) Objectives.

(a) Obtain data to determine if ATGM can track targets that are executing maneuvers thought to be limiting to the technical capabilities of the TOW, DRAGON, and Shillelagh systems.

(b) Obtain aiming error data to determine if tanks with conventional weapons systems can engage the same type of maneuvering targets.

(c) Provide data that may be used to:

1. Develop product improvements for existing antitank conventional and guided missile systems.
2. Develop follow-on requirements for antitank systems.
3. Improve antitank and armor crew programs for training.
4. Assist in improving existing combat simulation models.
5. Evaluate effects of target vehicle mobility on gunner tracking performance.

2. EXPERIMENTATION DESIGN DESCRIPTION.

a. The Field Test. The field test was conducted by CDEC at Fort Hunter Liggett, California during October-December 1975 in three subphases. The subphases corresponded to the amount of experimental control exercised over the target maneuver and were as follows:

(1) Subphase IIA was designed to evaluate gunner tracking when target vehicles performed exact imitations of the mathematically generated maneuvers selected for field testing in Phase I. Each path was visually marked for drivers of the target vehicles. Vehicle paths consisted of from two to four maneuvers connected by 50-meter straight stabilization segments. At no time was line of sight broken between the weapon systems and the target vehicles.

(2) Subphase IIB was the most operationally oriented approach to evaluating evasive maneuvers. Drivers of the target vehicles were in a free-play mode, which allowed them to utilize terrain and cover to break

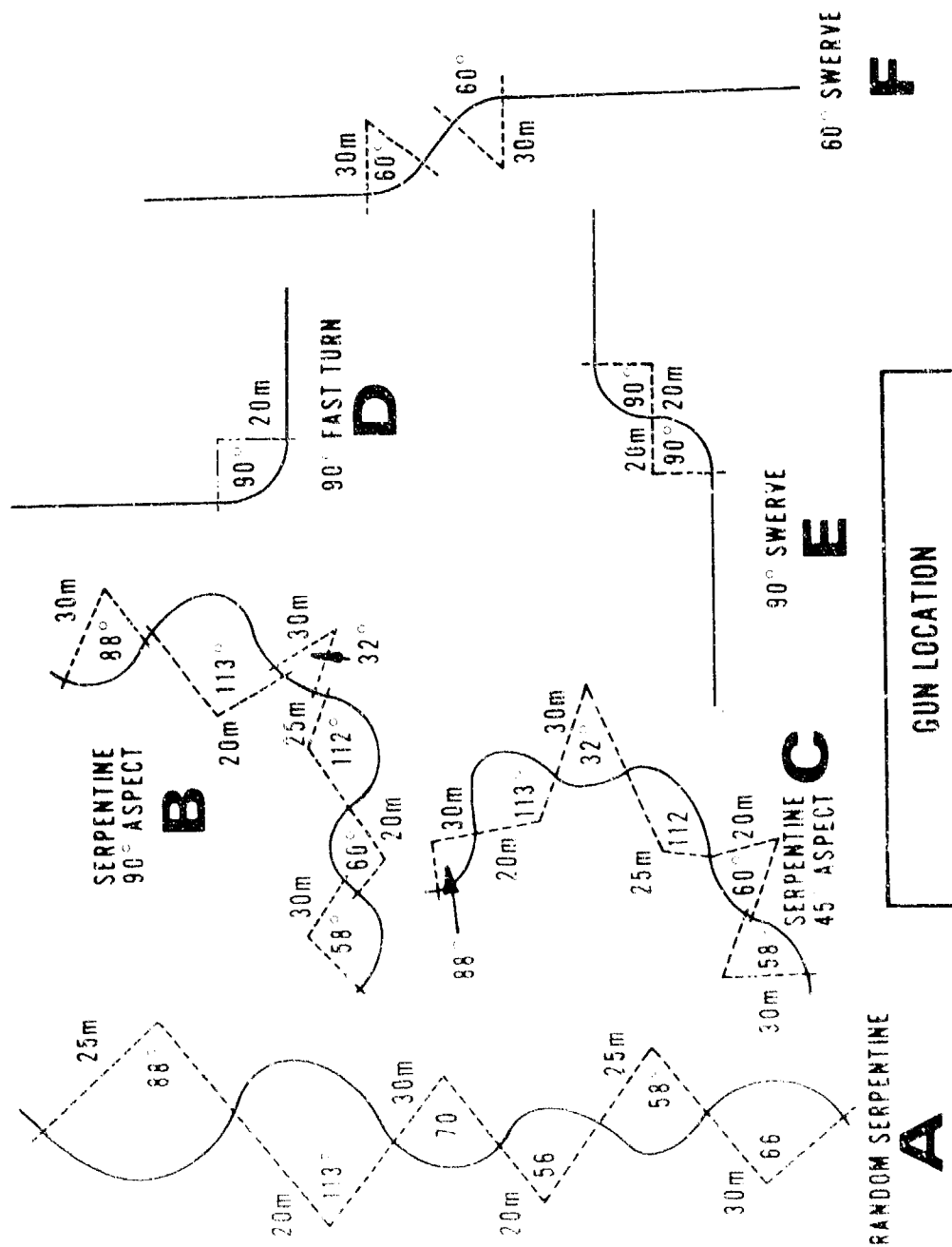


Figure 1. Phase II target maneuvers

line of sight while traversing the required terrain. Gunners indicated times at which they felt weapons should be fired.

(3) Subphase IIC employed the three maneuvers from Subphase IIA: the swerve, fast turn, and random serpentine. As in Subphase IIA, continuous line of sight was maintained and 50-meter straight stabilization segments between maneuvers were used. However, drivers were told the sequence of maneuvers to execute and the avenue of approach to take. They then performed their interpretation of the required maneuvers.

(4) Hereafter in this report, the subphases are referred to only as IIA, IIB, and IIC.

b. Design Variables.

(1) Controlled variables.

- (a) Width of maneuver path for IIA.
- (b) Fifty-meter stabilization segment between maneuvers for IIA and IIC.
- (c) Gunner skill level.
- (d) Gunner fatigue.
- (e) Target vehicle and driver performance.

(2) Uncontrolled variables.

- (a) Meteorological conditions.
- (b) Mechanical malfunctions.
- (c) Terrain.

(3) Independent variables.

(a) Antitank weapons systems: the DRAGON, TOW, Shillelagh mounted on both the M551 and the M60A2, and the M60A1 main gun.

(b) Range bands.

(c) Maneuvers.

(d) Target vehicles: M60A1 tank, XM800 Scout, and XM808 TWISTER.

(4) Dependent variables.

- (a) Gunner tracking.

- (b) Gunner firing events.
- (c) UTM coordinates of target vehicle.
- (d) Line-of-sight events.
- (e) Target vehicle elevation.

c. Instrumentation and Data Collection.

(1) Data collected during each of the subphases included the following:

(a) The X and Y coordinates of the target vehicle were collected at approximately 0.1 second intervals. Values were measured by a Range Measuring System (RMS), which determined the range of the maneuvering elements from known survey points and fed the data into a computer for position location calculations. The computer compared ranging data to the predicted location, based on previous pollings, to filter bad sensings. When polled data varied significantly from predicted locations, diagnostic messages were produced to permit invalidation. The nominal accuracy of this system is  $\pm 5$  meters. The RMS consisted of four cooperative elements. The A-stations interrogated the B-units (transponders) for messages and determined the slant ranges as a function of propagation time. One B-unit was mounted on each target vehicle. The C-station was the controlling element that determined what transmissions would take place and formatted the data into and out of the XFS930 computer. The D-station was a relay link between the C-stations and A-stations.

(b) Since no missiles were fired during the experiment, there were no missile sources to detect; therefore, to measure the gunner tracking error, the missile guidance system for each weapon system was used. The missile tracker recorded the displacement of the gunner's aim point from the infrared source mounted on each target vehicle. Because the infrared source was not collocated with the ideal aim point, corrections to the recorded displacement were required to eliminate this bias following the experiment. Data collection was accomplished with a modified Shillelagh trainer beacon detectable by antitank missile system trackers out to ranges beyond 3,000 meters. A two-position switch permitted rapid change from frequencies detectable by the Shillelagh system to those detectable by TOW and DRAGON systems.

(c) Administrative control data, such as trial identification, maneuver events, and start and stop times, were collected. This was accomplished by a voice recording system (VRS III), a self-contained recording, timing, and playback facility mounted in a climatically controlled semitrailer. This system provided the capability of recording up to 48 voice data channels on eight 7-channel tape recorders. The seventh channel on each tape deck recorded Interrange Instrumentation Group (IRIG) time, Format B. The time base was input from a Range Timing System (RTS),

synchronized with Radio Station WWB at Boulder, Colorado which radiates IRIG-B time throughout Fort Hunter Liggett.

(d) Calibration data needed to convert the gunner tracking error signal to angular error from the specified aim point was also obtained. These data consisted of readings taken at zero and plus and minus one mil deviations from the aim point.

(2) All data were time-tagged so that correlation of data could be made on a time basis. Additional data collected for IIB included times at which simulated firings occurred and times at which line of sight was interrupted, partially interrupted, and regained. This was accomplished by the VRS III and by locating a B-unit at the antitank weapon systems position and feeding all firing event data into that unit. After all data were collected, elevations corresponding to the X,Y coordinates of the target vehicles were obtained by cubic spline interpolation from aerial survey data for 10-meter grids.

(3) A more detailed and complete description of the field test procedures is available in the field test final report published by CDEC (reference 7).

### 3. SUBPHASE IIB OVERVIEW.

a. Background. Subphase IIB was designed to gather data under conditions similar to a tactical situation. Gunners were required to track an evasive target under intermittent LOS conditions and to indicate times at which they felt weapons should be fired. Each target vehicle driver was directed to follow a general avenue of approach but was allowed to maneuver freely and make maximum use of terrain. The only exception was that each driver was instructed to break LOS at least once during each trial for a minimum time period of 15 seconds. One-quarter of the trials were started with the target vehicle out of LOS. It was hoped that data collected under such operational conditions might yield additional insights into the anti-tank missile engagement process.

(1) Table 1 displays the Subphase IIB experimental design and the number of valid trials completed in each cell. Note that the same weapon systems and target vehicles are considered as in Subphases IIA and IIC. However, the four range bands were collapsed by combining range bands 1 and 2 and range bands 3 and 4. Trial sets then consisted of traversing terrain corresponding to these expanded range limits. The new range limits were 200 to 1,300 meters for range bands 1 and 2 and 1,600 to 3,000 meters for range bands 3 and 4.

(2) Figure 2 is a copy of a computer graphics display of the terrain, represented by 1,500-meter grid squares, on which all IIB trial sets were performed. Positions A and B mark the approximate locations of the weapons systems for range bands 1 and 2 and 3 and 4, respectively. The gunner positions were located on ridges overlooking the large central valley, and the target vehicles maneuvered in a southeasterly direction toward the

Table 1. Subphase IIB valid trials

Weapon System	Range Band	Target Vehicle		
		M60A1	XM800	Twister
DRAGON	1-2	26	16	9
	3-4	NA	NA	NA
M60A1	1-2	37	20	15
	3-4	36	33	15
TOW	1-2	31	20	16
	3-4	32	33	16
M60A2	1-2	25	15	7
	3-4	28	24	4
M551	1-2	20	15	6
	3-4	20	23	4



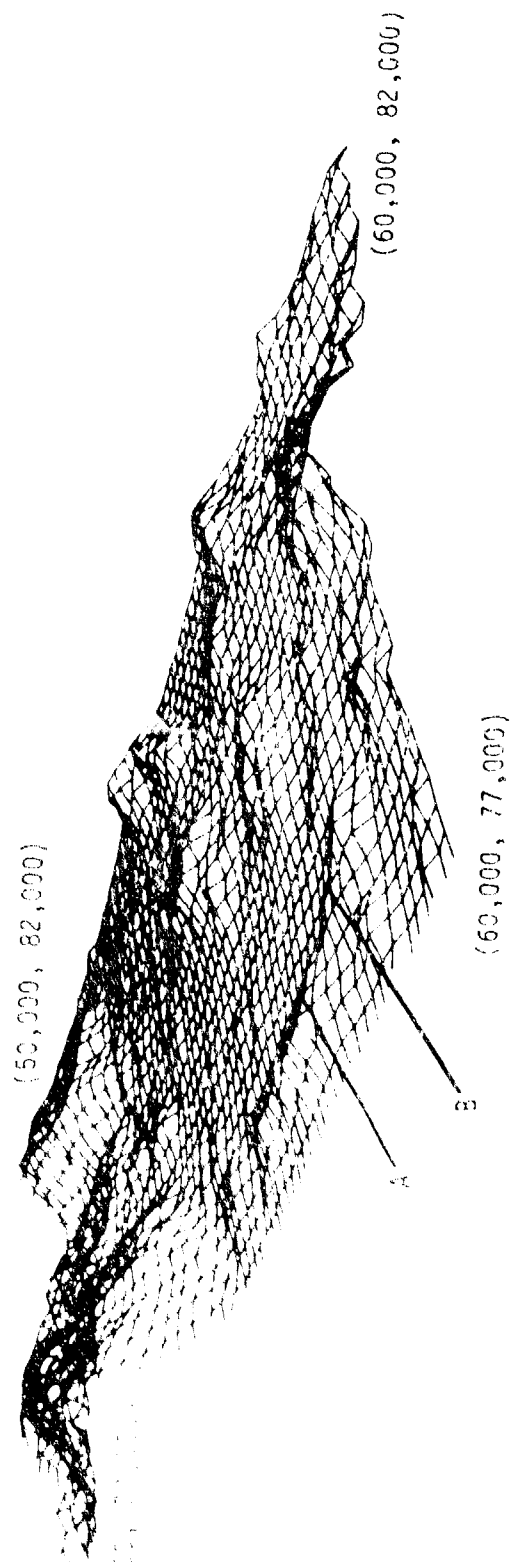


Figure 2. ATWT terrain site at Fort Hunter Liggett, California

weapon positions. Figure 3 displays the terrain as viewed from a TOW ATGM location at position A. The view from a TOW ATGM site at position B is portrayed in figure 4. The grid squares in both figures 3 and 4 are 50 meters each. Note that the elevation is exaggerated by a factor of 3.

b. Data Collection.

(1) Data regarding the target vehicle position-location, gunner tracking, and basic administration of the trial were collected as in Subphases IIA and IIC; but because IIB required the recording of gunner firing times and information concerning the intervisibility between weapon and target, additional data collection procedures were specified.

(a) Changes in LOS conditions were recorded by controllers, collocated with the ATGM systems, by generating a signal to the RMS B-units. Recorded signals indicated "no line of sight" (NLOS) when the vehicle was totally obstructed from view, "intermittent line of sight" or "intermittent vegetation" (IV) when the view of the vehicle was partially obstructed because of vegetation or terrain, and "line of sight" (LOS) when the vehicle was in unobstructed view.

(b) For the collection of firing data, the weapon systems' firing switches were wired directly to the RMS. Thus, when a gunner squeezed the trigger, a fire event was automatically recorded.

(2) Ideally, the analysis of the IIB data would consist of missile simulations using the actual gunner tracking record, the gunner firing times, and the target vehicle's recorded path. Such an analysis would yield an operational probability of hit subject only to the limited ability to generalize imposed by the specific terrain and tactical situation. However, a dilemma occurred in defining the tracking records of gunners because of the line-of-sight interruptions in IIB, significant difficulties were encountered early in the effort to digitize the analog gunner tracking data from this subphase. Large variations in the recording of tracking signals at the beginning and end of line-of-sight segments among gunners in the same trial were a major problem. Since the tracking signal relied on the antitank missile guidance system, rather than the gunner's optics, many causes could be hypothesized for such inconsistencies in the recording from one gunner to another. In the process of reacquiring a target vehicle following a break in line of sight, the gunner may have had the target in optical line of sight prior to having the target within view of the beacon-tracker system. Conversely, the gunner may have been receiving a signal from the beacon-tracking system prior to mentally perceiving that the vehicle was back in line of sight. Such inconsistencies, and the inability to distinguish among their origins, yielded data less than desirable for AMSAA's use in the antitank missile simulations. When LOS is lost, the beacon cannot be tracked, creating data gaps in the gunner tracking record. Without a continuous gunner tracking input, the simulation cannot be used. For this reason and by mutual agreement, AMSAA and CAIDA made the decision not to digitize analog data from IIB.

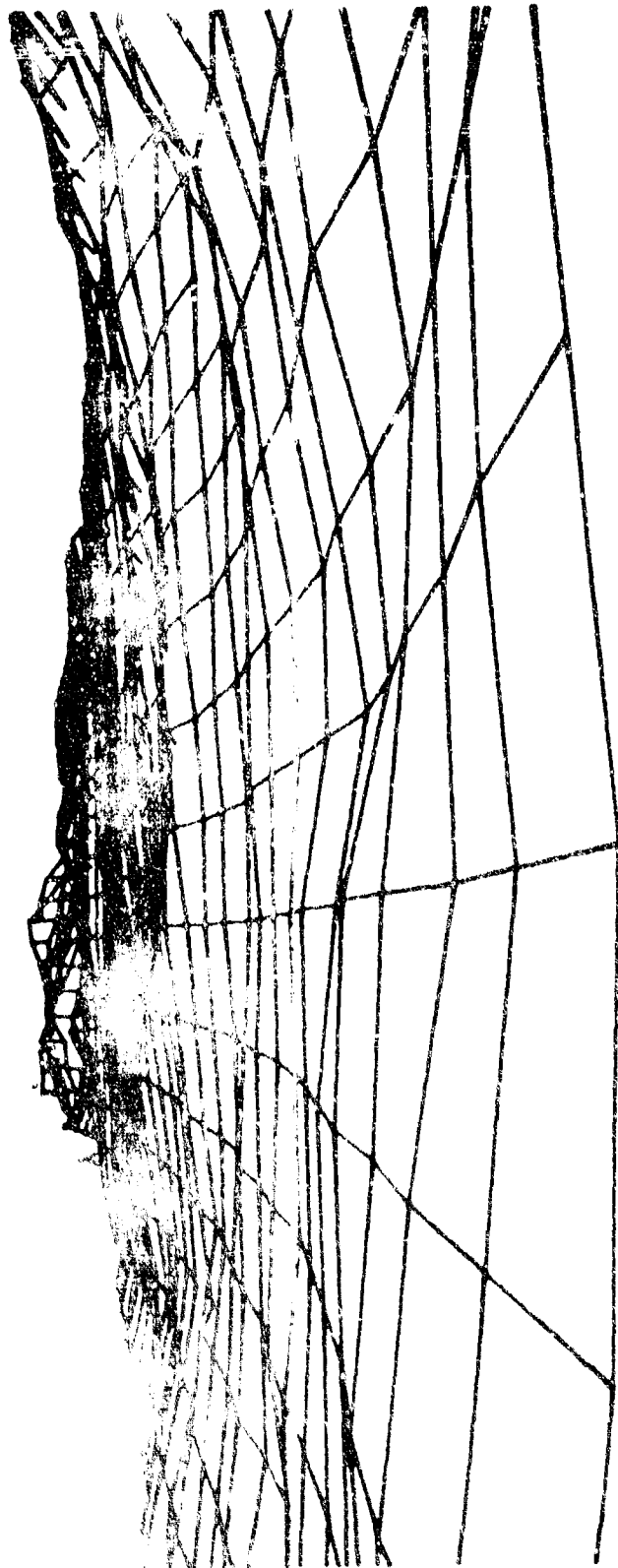


Figure 3. View from TOW ATGM location at position A

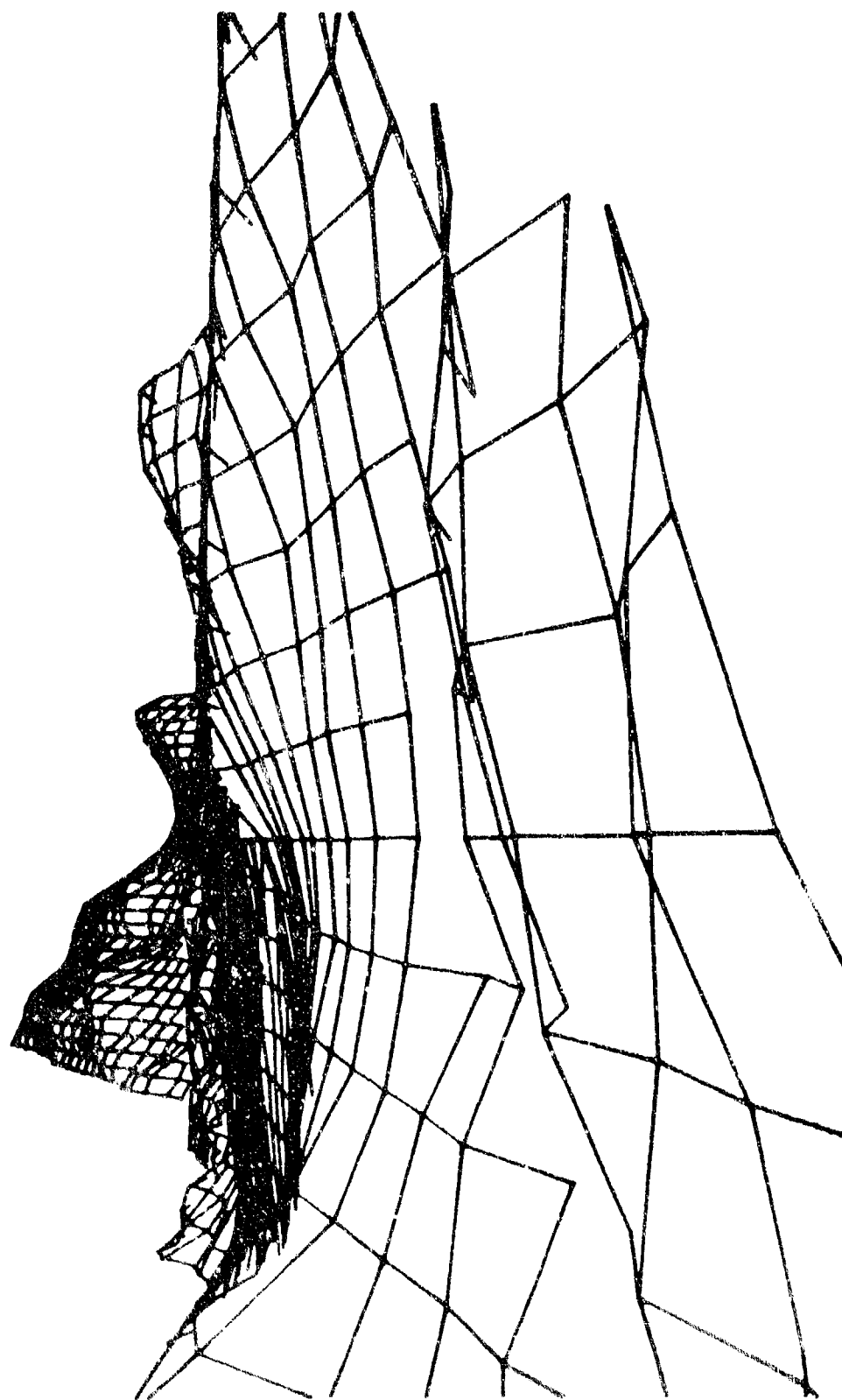


Figure 4. View from 13W ATGM location at position B

#### 4. RESULTS OF ANALYSIS.

a. Areas Analyzed. Since the missile simulations could not be performed because of the problems encountered with the tracking data, the IIB analysis was revised. This revision focused on three main efforts:

(1) An LOS analysis to determine the probability, displayed in the field experiment data, of successfully tracking the target vehicle from the simulated firing time to the projected impact time without losing LOS.

(2) An analysis of the velocities and accelerations of the target vehicles to portray their performance in the field experiment, for potential comparison against their projected capabilities.

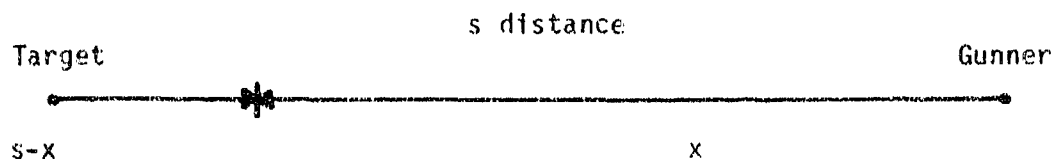
(3) An analysis of maneuver, velocity/acceleration, and raw tracking data to identify any possible observable correlations in that data.

b. Line-of-Sight Analysis. An analysis to determine the rate of missile aborts due to interruptions in LOS was performed. This was achieved by combining the LOS data reported by the controllers with gunner firing times and mathematically projecting missile flight times to obtain the number of missile aborts. This analysis indicates the probability for missile aborts for tactical situations and terrain that are similar to the test conditions. In addition, the results are used to access any possible weaknesses in gunner training and/or weapon system hardware.

##### (1) Methodology and results.

(a) Each time a missile firing event was simulated, a check was made to determine what LOS condition existed between the target vehicle and gunner at that moment. The existing LOS condition was recorded for further use. Possible LOS conditions at the firing time consisted of line of sight (LOS) and intermittent vegetation (IV). Any firings occurring when the controller reported no line of sight (NLOS) were deleted from the raw data, although this event occurred only once in 1,946 ATGM firings.

(b) The estimated time of flight (TOF) for the antitank missile to reach the target was computed utilizing information concerning the range of the target vehicle at firing, the rate at which the missile travels, and the target vehicle's rate toward the weapon system. The range at the firing time was interpolated linearly from range data generated every 5 seconds throughout the trial from the position-location data. The speed of the missile was determined from basic weapon system data provided by AMSAA. The vehicle's rate toward the weapon system was estimated from the distance the vehicle traveled during the time required for a missile to reach the vehicle at the estimated range at the firing time. Using a programed version of the algebraic logic displayed in figure 5, the TOF was estimated. The projected impact time was determined by adding the TOF to the firing time.



$s$  = distance between gunner and target at start of missile flight  
(estimated from 5-second RMS data)

$r_2$  = rate of missile (estimated from polynomial curve fitted to data with  
rate as a function of distance)

$r_1$  = rate of vehicle (estimated from distance traveled during missile  
flight time to maximum range)

$x$  = range to point of impact

NOTE: distance = rate  $\times$  time, and  $t_1 = t_2$ , assuming acceleration = 0.

Hence:  $d_1/r_1 = d_2/r_2$

Specifically:  $d_1 = s - x$

$$d_2 = x$$

$$(s - x)/r_1 = x/r_2$$

$$x = \frac{r_2 s}{(r_2 + r_1)}$$

Estimate time of flight from  $x$ :

$$\text{TOF} = x/r_2$$

Figure 5. Algebraic logic for missile time of flight estimation

(c) Once the projected impact times were estimated, a record was made of the LOS condition existing between the gunner and target vehicle for each projected impact time. Possible LOS conditions were LOS, IV, and NLOS. Combining the LOS conditions at missile firing and impact yielded six possible combinations of intervisibility. These are (1) LOS-LOS, (2) LOS-IV, (3) LOS-NLOS, (4) IV-LOS, (5) IV-IV, and (6) IV-NLOS. The frequency that each combination occurred was tabulated for total firings. In addition, categorizations by weapon system, range band, target vehicle, and combinations of these were assembled. Tables 2 and 3 display the number and percentages of intervisibility combinations occurring across all IIB trials and occurring by weapon system, respectively. Tables of the number of occurrences for other groupings are found in appendix B.

(d) For each trial the amount of time spent by the target vehicles in each LOS condition, LOS, IV, and NLOS, was determined. Based on the individual trial information, total time spent in each LOS condition was summed across trials. So that this information might be more useful, the amounts of time were changed to percentages of total time. Thus, the time spent in different LOS conditions could be compared with the percentage of simulated missile firings occurring for each firing and impact LOS condition. Arrays corresponding to those for the missile LOS firing and impact combinations were assembled. Tables 4 and 5 display the percentage of time spent by the target vehicles in each LOS condition and in each LOS condition isolated by weapon system, respectively. Other categories of LOS percentages are found in appendix C.

## (2) Discussion.

(a) Since the frequency of intervisibility combinations at ATGM firings and impacts may depend on the time spent in each LOS condition, a ratio analysis was undertaken to identify relationships between any independent variables and the LOS combinations. The three variables considered were weapon systems, range bands, and target vehicles. From the data directly and indirectly contained in the tables in appendixes B and C, two ratios were formed. These ratios may be considered as conditional probabilities. It is necessary to note that while these two ratios are useful for making comparisons within this subphase, they have no significant use outside of ATMT.

1. The first was determined by a comparison, as a quotient, of the probability of the LOS-LOS intervisibility combination with the probability of LOS occurring during the target vehicle maneuver. The LOS-LOS intervisibility combination refers to the availability of LOS from the gunner's position to the target at both trigger pull for the ATGM and at the projected time of missile impact. An implicit assumption is that LOS was maintained throughout missile flight when an LOS-LOS combination occurred. Symbolically, this ratio is expressed as:

$$R_c = \frac{\text{Prob (LOS-LOS)}}{\text{Prob (LOS)}} \quad (4.1)$$

Table 2. The number and percentage of LOS fire and impact inter-visibility combinations occurring across all Subphase IIB trials

LOS Condition	Number of Occurrences	Percentage
LOS - LOS	1601	82.31
LOS - IV	94	4.83
LOS - NLOS	85	4.37
IV - LOS	40	2.06
IV - IV	114	5.86
IV - NLOS	11	.57
TOTAL	1945	100.00



Table 3. The number and percentage of LOS fire and impact intervisibility combinations occurring by weapon system

LOS Condition	Weapon System							
	DRAGON		TOW		M551		M60A2	
	Number	Percent	Number	Percent	Number	Percent	Number	Percent
LOS - LOS	157	73.71	855	83.66	275	79.48	314	86.26
LOS - IV	16	7.51	42	4.11	21	6.07	15	4.12
LOS - MLOS	9	4.22	45	4.40	15	4.33	16	4.40
IV - LOS	8	3.76	15	1.47	13	3.76	4	1.10
IV - IV	23	10.80	60	5.87	17	4.91	14	3.85
IV - MLOS	0	0.00	5	.49	5	1.45	1	.27
TOTAL	213	100.00	1022	100.00	346	100.00	364	100.00

Table 4. The percentage of total trial time spent by the target vehicles in each LOS condition across all Subphase IIB trials

LOS Condition	Percentage of Time
LOS	61.64
IV	18.40
NLOS	19.96
TOTAL	100.00

Table 5. The percentage of total trial time spent by the target vehicles in each LOS condition as observed by each weapon system

LOS Condition	Weapon System			
	DRAGON	TOW	M551	M60A2
LOS	69.30	65.94	55.74	56.29
IV	16.41	16.33	19.62	21.52
NLOS	14.29	17.73	24.64	22.19
TOTAL	100.00	100.00	100.00	100.00

This measure yields the relative number of missiles potentially completing their flight to the target area per percentage unit of target vehicle LOS time existing during a maneuver. Hereafter, this ratio is referred to as the missile completion ratio ( $R_c$ ).

2. The second ratio represents the quotient of the sum of the probabilities for the LOS-NLOS and IV-NLOS intervisibility combinations compared with the probability of NLOS.

$$R_a = \frac{\text{Prob (LOS-NLOS)} + \text{Prob (IV-NLOS)}}{\text{Prob (NLOS)}} \quad (4.2)$$

This ratio expresses the relative number of probable missile aborts per percentage unit of target vehicle NLOS time existing. A missile abort was considered to have occurred when the target vehicle could not be seen at the projected time of missile impact. Hereafter, this ratio is referred to as the missile abort ratio ( $R_a$ ).

(b) When the differences in the size and pattern of these ratios across the different combinations of independent variables are examined, three results are visually evident.

1. The target vehicle used has no significant effect on either ratio.

2. The value of the missile abort ratio increases when going from range bands 1 and 2 to range bands 3 and 4. As range increases, there is a corresponding TOF increase for each missile and, thus, an increase in Prob (LOS-NLOS) and Prob (IV-NLOS) combinations. Consequently, more missile aborts occur at long range than at short range.

3. When the missile completion ratio for each weapon system is considered, the ratio decreases from a high for the Shillelagh (M60A2) to the Shillelagh (M551) to the TOW to a low for the DRAGON. This seems to indicate that the relative effectiveness of flying a missile to the target area is greatest for the M60A2 system and lowest for the DRAGON, although no definite cause has been identified for this result. Examination of both numerators and denominators of these ratios reveals no consistent trends, but the order and magnitude differences of the resulting ratios are consistent throughout the combinations of independent variables. It is possible that an undetermined bias was present in either the testing or the data collection process. This is conceivable because the Shillelagh systems (M60A2 and M551) were tested in one set of trials and the TOW and DRAGON systems were tested in another trial set. Other possible explanations for the  $R_c$  differences may include gunner training, differences in the field of view among the ATGM systems, a combination of these, or some still undefined parameters.

(c) It should be noted that this report considers only the probability that LOS is maintained throughout the missile TOF. Another intervisibility problem, which precedes the problem considered here,

involves the frequency and duration of LOS needed to detect a maneuvering target and start the engagement processes. Data to consider this problem are not available from ATMT, but it was considered previously in the TETAM report (reference 8). The probability that an antitank missile destroys an opposing target can be thought of as a series of several conditional probabilities. One formulation of such a series is as follows:

$$P(K) = P(D) \cdot P(A/D) \cdot P(T/A) \cdot P(LOS/T) \cdot P(H/LOS) \cdot P(K/H)$$

Where:  $P(K)$  is probability of kill

$P(D)$  is probability of detection

$P(A/D)$  is probability of acquisition, given detection

$P(T/A)$  is probability of trigger pull, given acquisition

$P(LOS/T)$  is probability of LOS until the missile reaches the target, given trigger pull

$P(H/LOS)$  is probability of hit, given LOS during missile flight

$P(K/H)$  is probability of kill, given a hit.

In the ATMT analysis provided here, only  $P(LOS/T)$  is determined and then only for one specific set of terrain. It becomes obvious that the probability of kill is dependent on several important conditions and if any one condition has a low probability the chance for a target kill is seriously degraded.

(d) Based on the data collected in TETAM, Johnsrud and Shedowski (reference 9) computed the probability of TOW and DRAGON missile aborts during missile flight time due to terrain and vegetation masking. By fitting the TETAM segment length distributions to an exponential form, they were able to compute the probability that an occurring segment length would exceed a given length. Using information from such distributions with parameters unique to the terrain considered (i.e., range to target, missile velocity, and attacker (target) velocity), they computed the probable percentage of missile aborts occurring during missile flight. Since TETAM site A at Fort Hunter Liggett and the ATMT trial area nearly coincide, it is appropriate to compare the abort probabilities obtained in ATMT with the statistically derived probabilities of Johnsrud and Shedowski. The two methods show fairly good agreement; however, there is a tendency for the ATMT abort values to be slightly lower. This might be attributed to the ability of the gunners in ATMT to anticipate changes in LOS and refrain from firing in obvious abort situations, or from the slightly different locations for gunners and the approach paths of the target vehicles.

(4) Findings.

(a) Based on the information contained in tables 2 and 3 and in appendix B, the relative frequency of an ATGM gunner having line of sight from the time a missile firing was recorded until the projected time of impact at the range of the target vehicle varied from a low of 64.70 percent for trials involving the TOW ATGM against the XM808 TWISTER in range bands 3 and 4 to a high of 95.45 percent for Shillelagh (M60A2) trials against the TWISTER in range bands 1 and 2.

(b) The relative amount of time the target vehicles spent in LOS with respect to each group of ATGM positions varied from a low of 49.47 percent for the TWISTER in range bands 3 and 4 as viewed from the TOW positions to a high of 71.26 percent for the XM800 Scout in range bands 1 and 2 as viewed from both the TOW and DRAGON positions.

(c) An increase in the number of expected missile aborts because of changes in LOS conditions was evident as the range to the target vehicles increased. This was expected because as the range increased, TOF of the missiles increased; hence, there was a greater likelihood that the ATGM gunners would not be able to anticipate projected LOS changes at the time of missile firing.

(d) Although no confirmed explanation can be given, the relative effectiveness of the weapon systems in completing missile flights per unit of time were, in descending order: the Shillelagh (M60A2), Shillelagh (M551), TOW, and DRAGON.

c. Target Vehicle Velocities and Accelerations. From the data collected by the range measuring system (RMS), it is possible to estimate the velocities and accelerations for the target vehicles. Such data serve as indicants of the maneuverability of the target vehicles. Since the obtained data came from a test where drivers were told to maneuver realistically at the maximum safe speed on actual terrain, the actual field capabilities of the target vehicles may be inferred from the results obtained.

#### (1) Methodology and results.

(a) For each trial performed in IIB, the average resultant velocity and acceleration, as well as x,y,z-components for the target vehicle, were computed using the UTM coordinate system. These values were obtained by averaging estimates of the appropriate parameters throughout each trial. Estimates were obtained at approximately 0.1-second intervals. Because the raw data contained inherent measurement errors, a smoothing procedure was necessary to obtain estimates lying within realistic bounds for velocity and acceleration. A moving 35-point least squares cubic polynomial (appendix D) was used to smooth the raw data. The estimates obtained in the UTM coordinate system were combined with range to provide angular rate and angular acceleration from the gunner's point of view. Also, the UTM, x,y,z-components for velocity and acceleration were rotated to x,y,z-components where the y-axis reflected the gunner-target line. This

provided information concerning the motion of the target vehicle toward the gunner and the remaining motion the gunner would perceive in the xz-plane.

(b) The results obtained must be evaluated in view of one overriding condition. Each driver was told to remain out of LOS for a minimum of 15 continuous seconds in each trial. In many cases, the driver simply found a position he knew was not in LOS and stopped his vehicle. Because the effects of decelerating and accelerating from such stops could not be removed from the data, estimates obtained immediately preceding, during, and following such stops were included in the averages. In view of this, the results presented in tables 7 through 13 may be considered slightly low for a target vehicle that would be in continuous motion. Future test designs should preclude occurrences of this nature. However, in no trial did 15 seconds exceed 10 percent of the trial time reflecting vehicle motion.

(c) Tables 6 and 7 present the means and standard deviations of the resultants and components in the UTM coordinate system. Note that for velocity 1 meter per second equals 2.237 miles per hour and for acceleration 1 gravitational unit equals 9.806 meters per second squared. In tables 6 through 13 the averages and standard deviations reflect that the estimates at each point were weighted equally across trials.

(d) Tables 8 and 9 indicate the average velocities and accelerations rotated so that the y-axis serves as the gunner-target line. Thus, the y-component in table 8 reflects the average speed at which the target vehicle is approaching the gunner. Disregarding the change in size of target vehicle as it approaches the gunner, the xz-velocity and acceleration resultants indicate both the target motion the gunner must adjust to and its rate of change.

(e) Tables 10 through 13 present the average angular rate and angular acceleration for the two range bands of IIB. Comparison of the averages, and especially the standard deviations, demonstrates how the angular motion the gunner must perceive and account for in the aiming of his weapon increases as the target approaches his position.

## (2) Findings.

(a) As expected, the velocity and acceleration data indicate that the TWISTER was able to maneuver with the highest velocity and with the most rapid changes in that velocity. The Scout and M60A1 followed, in that order.

(b) Based on the means, the average cross-country speed displayed in IIB ranges from approximately 11 miles per hour for the M60A1 to 16 miles per hour for the TWISTER. Hypothesizing that the maximum speed is in the neighborhood of 2.5 standard deviations above the mean, maximum speeds vary from about 25 miles per hour for the M60A1 to 35 miles per hour for the TWISTER.

Table 6. Target vehicle velocity in the UTM coordinate system: means and standard deviations

Vehicle Type	Number of Points	Velocity (m/sec)		
		Resultant	X-Component	Y-Component
HECATE	122,946	5.105 (2.305)	3.535 (2.643)	-.565 (3.341)
Scout	66,047	6.414 (2.845)	4.495 (3.388)	-.935 (3.814)
TRISTER	24,657	6.982 (3.236)	5.281 (3.945)	-.895 (3.776)
Overall	224,550	5.701 (2.593)	4.013 (3.044)	-.712 (3.537)
				.023 (.259)



Table 7. Target vehicle acceleration in the UTM coordinate system: means and standard deviations

Vehicle Type	Number of Points	Acceleration (m/sec <sup>2</sup> )			
		Resultant	X-Component	Y-Component	Z-Component
M60A1	132,934	.911 (.657)	-.001 (.667)	.002 (.905)	.000 (.124)
Scout	66,936	1.342 (1.024)	-.004 (.995)	.002 (1.341)	.000 (.164)
THUNDER	24,607	1.501 (1.206)	.005 (1.175)	-.001 (1.527)	.000 (.165)
Overall	224,517	1.104 (.853)	-.002 (.943)	.002 (1.130)	.000 (.142)



Table 3. Target vehicle acceleration in a rotated coordinated system: means and standard deviations

Vehicle Type	Number of Points	Acceleration (m/sec <sup>2</sup> )				
		Resultant	XZ Resultant	X-Component	Y-Component	Z-Component
M60A1	132,934	.911 (.657)	.678 (.604)	.002 (.913)	.006 (.661)	.000 (.125)
Scout	66,936	1.342 (1.024)	1.021 (.945)	.003 (1.381)	.006 (.995)	.000 (.161)
THISTER	24,647	1.501 (1.206)	1.100 (1.067)	.001 (1.532)	.006 (1.168)	.000 (.165)
Overall	224,517	1.104 (.853)	.827 (.779)	.002 (1.149)	.006 (.839)	.000 (.141)

Table 10. Target vehicle angular velocity in range bands 1 and 2: means and standard deviations

Vehicle Type	Number of Points	Velocity (mils/sec)	
		Azimuth	Elevation
M60A1	60,615	-.566 (4.151)	-.071 (.414)
Scout	20,814	-.621 (5.334)	-.095 (.488)
TWISTER	10,897	-.671 (5.271)	-.123 (.481)
Overall	92,326	-.591 (4.583)	-.083 (.440)

Table 11. Target vehicle angular acceleration in range bands 1 and 2: means and standard deviations

Vehicle Type	Number of Points	Acceleration (mils/sec <sup>2</sup> )	
		Azimuth	Elevation
M60A1	60,611	-.004 (1.160)	.002 (.220)
Scout	20,814	.021 (1.938)	.001 (.319)
TWISTER	10,897	.027 (2.115)	.002 (.319)
Overall	92,322	.005 (1.503)	.002 (.258)

Table 12. Target vehicle angular velocity in range bands 3 and 4: means and standard deviations

Vehicle Type	Number of Points	Velocity (mils/sec)	
		Azimuth	Elevation
M60A1	72,331	-.253 (1.348)	-.044 (.102)
Scout	46,133	-.340 (1.479)	-.050 (.109)
TWISTER	13,760	-.294 (1.499)	-.056 (.110)
Overall	132,224	-.288 (1.411)	-.047 (.105)

Table 13. Target vehicle angular accelerations in range bands 3 and 4: means and standard deviations

Vehicle Type	Number of Points	Acceleration (mils/sec <sup>2</sup> )	
		Azimuth	Elevation
M60A1	72,323	.002 (.403)	.000 (.061)
Scout	46,122	.002 (.571)	.000 (.059)
TWISTER	13,750	-.001 (.650)	.000 (.061)
Overall	132,195	.002 (.497)	.000 (.060)

(c) Again, based on the means, the average acceleration ranges from 0.09g for the M60A1 to 0.15g for the TWISTER. Utilizing 2.5 standard deviations above the mean to estimate maximum accelerations, the values range from 0.25g for the M60A1 to about 0.5g for the TWISTER.

d. Relationship of Maximum Gunner Error with Target Vehicle Motion.  
By considering the data available concerning target vehicle paths, velocities, and accelerations and the data available describing gunner errors, the Subphase IIB data were examined for detectable relationships among relevant gunner error variables and target vehicle variables. Since the gunner error data from this subphase were not digitized, the resulting analysis was severely limited in scope. However, any relationships identified could point out needs for further testing and/or more detailed and sophisticated analysis. In addition, if predominant relationships are displayed in the data, such relationships might become the basis for operational recommendations to target vehicle drivers and their training process and/or to ATGM gunners and their training process.

(1) Methodology and results.

(a) Although the gunner tracking data for this subphase were not digitized, the strip charts of gunner tracking performance were available for analysis. Figure 6 displays a portion of a typical ATMT strip chart. The lowest band records time in Interrange Instrumentation Group (IRIG) format, and the upper bands (two per gunner) record azimuth and elevation error for each of the possible gunners. For a sample of trials and for each LOS segment within these trials, the time to the nearest 0.5 second, the maximum azimuth error, and the maximum elevation error occurring for each gunner were determined manually from the strip charts. Except for the cells with the TWISTER against the M60A2/M551 where only two and one trial sets were available, the sample was constructed by picking three trial sets per design cell (table 14). Thus, gunner error maximums from 33 trial sets were examined. This resulted in 1,108 sample values for maximum azimuth error and 1,073 sample values for maximum elevation error. Table 15 presents descriptive statistics regarding the sample of observed maximum errors and their absolute values.

(b) Using time-correlated plots of vehicle maneuver and the times at which the maximum errors were observed, the vehicle maneuvers occurring approximately 0 to 3 and 0 to 12 seconds prior to maximum error were classified according to an alphanumeric scheme. This scheme consisted of an alphabetic character indicating the general type of maneuver followed by a numeric character indicating the aspect of this maneuver to the gunner's position. Details of this scheme are displayed in figure 7. The frequency of occurrence for each type of maneuver displayed is depicted in table 16. The surprising statistic from this table is the frequency with which maximum errors were observed when the target vehicle was moving directly toward the gunner.

(c) In addition to viewing time-correlated plots of target vehicle movement, the velocities, accelerations, and their components were



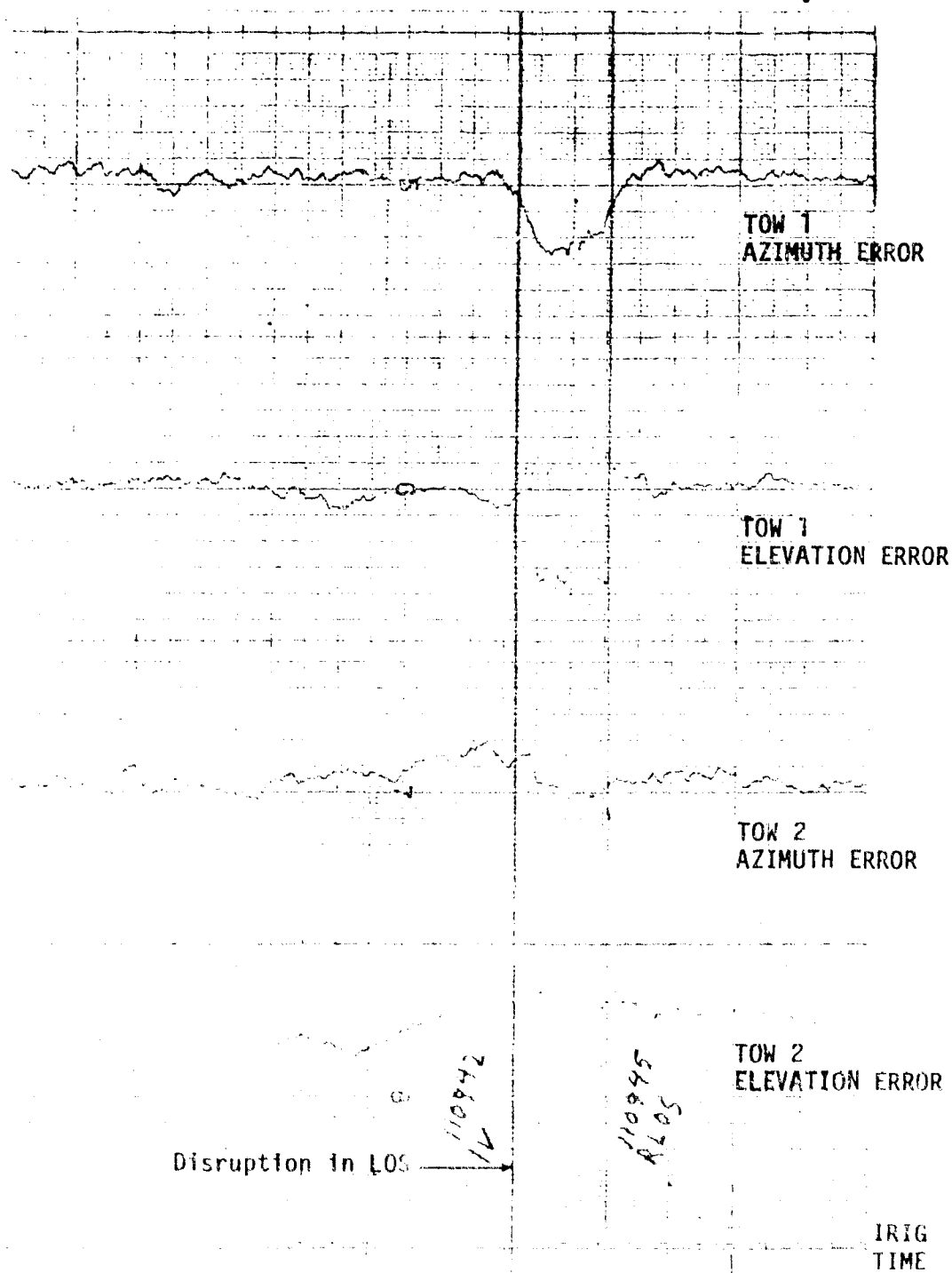


Figure 6. TOW gunner tracking error sample

Table 14. Phase IIB design matrix

Target	DRAGON/TOW/M60A1		M60A2/M551	
	Range Band		Range Band	
	1-2	3-4	1-2	3-4
SCOUT (XM800)	319 320* 327 413* 440*	311 312 321 322 412* 427* 428 441* 442	361* 371* 372* 436	366* 367 369* 370 417A 439*
M60A1 (TANK)	307* 308 325 326* 414 430 431 435* 436 444	302 309 310 407 425* 426* 432 433 434 443*	353 354 363* 364 405* 406* 422 423 424	349 350 355 356 403* 404 419* 420 421*
TWISTER (XM808)	315* 316* 317 411*	314* 323* 410* 429	357* 358*	352*

\* Indicates trial set used in sample to collect maximum gunner error data.

Table 15. Observed maximum gunner errors, azimuth and elevation

	<u><math>\bar{X}</math></u>	<u>SD</u>	<u>Min</u>	<u>Max</u>	<u>Skew</u>	<u>Kurtosis</u>
Azimuth error (meters)	-.237	2.820	-10.41	14.140	.246	1.388
Azimuth error (mils)	-.175	1.655	-3.950	7.490	.170	-.699
Absolute azimuth error (meters)	2.282	1.671	.20	14.140	2.014	6.529
Absolute azimuth error (mils)	1.497	.726	.07	7.49	1.240	5.143
Elevation error (meters)	.380	2.248	-6.97	8.14	.077	-.201
Elevation error (mils)	.197	1.488	-3.460	4.090	-.122	-.759
Absolute elevation error (meters)	1.929	1.213	.06	8.14	1.344	2.498
Absolute elevation error (mils)	1.316	.721	.05	4.09	.903	.675



Table 16. Frequency of maneuver occurring prior to maximum gunner error

Code	AZMAN3		AZMAN12		ELMAN3		ELMAN12	
	Freq	Rel Freq	Freq	Rel Freq	Freq	Rel Freq	Freq	Rel Freq
A1	13	1.2	70	6.3	20	1.9	83	7.7
B1	1	0.1	0	0.0	0	0.0	0	0.0
B2	0	0.0	0	0.0	0	0.0	2	0.2
C1	3	0.3	10	0.9	0	0.0	6	0.6
C2	0	0.0	7	0.6	0	0.0	7	0.7
C3	6	0.5	19	1.7	2	0.2	17	1.6
C4	7	0.6	1	0.1	7	0.7	6	0.6
D1	48	4.3	17	1.5	35	3.3	19	1.8
D2	93	8.4	25	2.3	80	7.5	31	2.9
D3	49	4.4	41	3.7	40	3.7	33	3.1
D4	43	3.9	31	2.8	50	4.7	39	3.6
E1	0	0.0	23	2.1	0	0.0	23	2.1
E2	1	0.1	27	2.4	0	0.0	27	2.5
F1	89	8.0	69	6.2	91	8.5	59	5.5
F2	75	6.8	51	4.6	71	6.6	48	4.5
F3	57	5.1	64	5.8	56	5.2	63	5.9
F4	77	6.9	57	5.1	80	7.5	63	5.9
G1	297	26.8	338	30.5	311	29.0	303	28.2
G2	49	4.4	51	4.6	59	5.5	43	4.0
G3	71	6.4	92	8.3	62	5.8	93	8.7
G4	129	11.6	115	10.4	109	10.2	108	10.1
Total	1108	100.0	1108	100.0	1073	100.0	1073	100.0

AZMAN3: Maneuver occurring 0-3 seconds prior to maximum azimuth error.

AZMAN12: Maneuver occurring 0-12 seconds prior to maximum azimuth error.

ELMAN3: Maneuver occurring 0-3 seconds prior to maximum elevation error.

ELMAN12: Maneuver occurring 0-12 seconds prior to maximum elevation error.

determined at times corresponding to maximum gunner errors. Specifically, the maximum velocity and acceleration occurring during the interval from 0.50 seconds preceding to 0.25 seconds following maximum gunner error were determined. A 0.75-second interval was used because maximum error times were recorded to the nearest 0.5 second, and it was felt that if target velocities and accelerations were to increase gunner error, those vehicle characteristics causing the increase would slightly precede the displayed error. After the maximum velocity and acceleration were found in the interval for each occurrence of maximum gunner error, their x-, y-, and z-components with respect to the gunner positions were determined. Table 17 displays average resultant velocity and acceleration data, which correlate with the times for maximum gunner errors. When these results are compared with those in tables 6 and 7, one can see a significant difference between the resultant velocities and accelerations occurring in the gunner error correlated and overall vehicle performance data. However, this significance must be noted remembering that the gunner error correlated values were a maximum value within a 0.75-second interval.

(d) Using the different measurements of maximum gunner error as dependent variables and velocity and acceleration components, as well as range, as independent variables, 16 stepwise linear regressions were examined for a relationship between maximum gunner error and target vehicle performance. Table 18 indicates the dependent and independent variables used in each of the 16 regressions. The stepwise regressions were performed using the Statistical Package for the Social Sciences (SPSS) computer software. For the final variables in the regressions equations, table 19 displays the multiple correlation, R, and the proportion of dependent variable variance accounted for by the independent variables,  $R^2$ . Independent variables were entered into the regression equation until the SPSS default values were reached. The default values allow for all but insignificant variance to be entered into the equations. As can be seen from this table, the absolute values of the maximum errors correlate much more highly with the independent variables than do the maximum errors. The highest percent of variance being accounted for amounts to 37.1 percent in regression equation 4. A breakdown of this equation is presented in table 20. As one can see, range is the first variable entered into the regression equation and accounts for a large portion of variance. In all other regression equations involving absolute values of maximum gunner error, range is always the first variable entered into the regression equation and accounts for most of the explained variance. This result implies that the range to the target is more significant than target vehicle velocity and acceleration components in predicting the absolute value of maximum gunner error. Since only fair results were noted in predicting absolute maximum gunner errors and extremely poor results were seen in predicting maximum gunner error, the analysis was terminated with the conclusion that target vehicle velocities and accelerations have little correlation with maximum gunner error.

## (2) Findings.

Table 17. Average velocity and acceleration at maximum gunner error\*

	$\bar{X}$	SD	N
Velocity (m/sec) at maximum azimuth error	6.588	2.607	1103
Acceleration (m/sec <sup>2</sup> ) at maximum azimuth error	1.532	1.037	1103
Velocity (m/sec) at maximum elevation error	6.546	2.497	1070
Acceleration (m/sec <sup>2</sup> ) at maximum elevation error	1.510	1.052	1070

\* Values taken in the .75 second interval about the maximum error.

Table 18. Regression analysis dependent and independent variable combinations\*  
(continued next page)

Regression No.	Dependent Variables								Independent Variables										
	A	B	C	D	E	F	G	H	$\dot{X}$	$\dot{Y}$	$\dot{Z}$	$\dot{\phi}$	$\dot{\theta}$	$\bar{X}$	$\bar{Y}$	$\bar{Z}$	$\bar{\phi}$	R	
1	X								X	X	X			X	X	X			X
2		X							X	X	X			X	X	X			X
3			X						X	X	X			X	X	X			X
4				X					X	X	X			X	X	X			X
5	X											X	X						
6		X										X	X				X	X	X
7			X									X	X				X	X	X
8				X								X	X				X	X	X
9					X				X	X	X			X	X	X			X
10						X				X	X			X	X	X			X
11							X			X	X			X	X	X			X
12								X		X	X			X	X	X			X
13					X							X	X				X	X	X
14						X						X	X				X	X	X
15							X					X	X				X	X	X
16								X				X	X				X	X	X

\* Headings defined on the following page.



Table 18. Regression analysis dependent and independent variable combinations (concluded)

DEPENDENT VARIABLES

- A: Azimuth error (meters)
- B: Azimuth error (mils)
- C: Absolute value of azimuth error (meters)
- D: Absolute value of azimuth error (mils)
- E: Elevation error (meters)
- F: Elevation error (mils)
- G: Absolute value of elevation error (meters)
- H: Absolute value of elevation error (mils)

INDEPENDENT VARIABLES

- $\dot{X}$ : x-component of velocity (m/sec)
- $\dot{Y}$ : y-component of velocity (m/sec)
- $\dot{Z}$ : z-component of velocity (m/sec)
- $\dot{\alpha}$ : Azimuth component of velocity (mils/sec)
- $\dot{\phi}$ : Elevation component of velocity (mils/sec)
- $\ddot{X}$ : x-component of acceleration (m/sec<sup>2</sup>)
- $\ddot{Y}$ : y-component of acceleration (m/sec<sup>2</sup>)
- $\ddot{Z}$ : z-component of acceleration (m/sec<sup>2</sup>)
- $\ddot{\alpha}$ : Azimuth component of acceleration (mils/sec<sup>2</sup>)
- $\ddot{\phi}$ : Elevation component of acceleration (mils/sec<sup>2</sup>)
- R: Range to target vehicle (meters)

Table 19. Multiple correlation and the explained variance resulting from the regression equations

<u>Regression No.</u>	<u>R multiple correlation</u>	<u>R<sup>2</sup>, proportion of variance accounted for</u>
1	.131	.017
2	.114	.013
3	.325	.105
4	.609	.371
5	.140	.020
6	.064	.004
7	.329	.108
8	.602	.362
9	.150	.022
10	.175	.031
11	.454	.206
12	.564	.318
13	.188	.035
14	.143	.020
15	.455	.207
16	.557	.310

Table 20. Stepwise breakdown of regression equation number 4

Step	Variable	Coefficient	Multiple R	$R^2$	Increment in $R^2$
1	Range	.00106	.60127	.36152	.36152
2	Velocity y-component	-.03825	.60547	.36659	.00507
3	Acceleration y-component	-.07363	.60708	.36855	.00196
4	Acceleration z-component	.35686	.60817	.36987	.00132
5	Velocity z-component	.17063	.60873	.37056	.00069
6	Acceleration x-component	.01916	.60899	.37087	.00031
7	Velocity x-component	-.00440	.60908	.37097	.00010
	(Constant)	.59943			

(a) An unexpected number of maximum errors occurred while the target vehicles were moving in a straight line path, especially toward the gunner.

(b) Average resultant velocities and accelerations time correlated with maximum gunner errors are greater than corresponding average overall trial velocities and acceleration for target vehicles. It must be remembered, however, that the time correlated values were maximums occurring within a 0.75-second interval surrounding the time of maximum gunner error.

(c) Maximum gunner errors demonstrate only a slight multiple linear correlation with a target vehicle's velocity and acceleration components, and its range. When the absolute values of gunner error are considered, moderate multiple correlations are observed. Noting these results, the relationship between gunner error and target vehicle motion does not appear to warrant further investigation without at least near continuous gunner and vehicle input and a more appropriate time series analysis technique.

## 5. CONCLUSIONS.

### a. Line-of-Sight Analysis.

(1) A maneuvering target vehicle that utilizes terrain and vegetation characteristics has the capability to impose an appreciable number of missile abort situations. The frequency and duration of target vehicle LOS interruptions, density of the vegetation, and range from the target to the gunner are influential parameters in producing these missile abort situations.

(2) TOW gunners were best able to perceive the probability of an LOS-LOS combination occurring in the firing and impact sequence at ranges less than 1,600 meters. As the range increased, Shillelagh gunners from both the M551 and M60A2 had the best perception of the LOS-LOS probability when firing.

(3) It appeared that the XM808 TWISTER was the target vehicle that most successfully disrupted the LOS-LOS combination of the DRAGON and TOW gunners firing and impact sequences. The Shillelagh gunners, from both the M551 and M60A2, had more difficulty achieving the LOS-LOS combination against the XM800 Scout than against the other two target vehicles.

### b. Target Velocities and Accelerations.

(1) The perspective and range from which a gunner observes a maneuvering target significantly affects the amount of target motion presented to the gunner.

(2) The average vehicle velocities and accelerations demonstrated on cross-country terrain are lower than might be expected considering performance standards stated for the vehicles.

(3) The differences among mean velocities and accelerations for the different target vehicles are not as pronounced as might be expected.

c. Relationship of Maximum Gunner Error with Target Vehicle Motion.

(1) Under operational conditions and with the analysis techniques used, target vehicle velocities and accelerations show little relationship with large gunner errors.

(2) No maneuver appears to be any more significant at causing major errors in gunner tracking than any other maneuver. In fact, maneuver alone cannot be considered as a cause for significant gunner error.

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## APPENDIX A

### SELECTION OF CANDIDATE MANEUVERS

A-1. Figure A-1 depicts the missile flight path from tracker to target, with miss distance caused by both gunner error and missile error. Tables A-1 through A-3 present the course parameters and geometry of each of the target maneuvers and figures A-2 through A-12 illustrate the actual maneuvers investigated.

A-2. MANEUVERS INVESTIGATED. Nine types of maneuvers were investigated, with several variations within each type. The general types are as follows:

a. START: From a stationary position the target accelerates in a direction normal to the LOS.

b. STOP: Traveling normal to LOS at fixed speed, the target decelerates to a stop.

c. STOP/START: Types a and b combined.

d. FAST TURN: Traveling at constant speed, the target executes a  $90^{\circ}$  turn, continuing in new direction.

e. S TURN: Same as type d except two  $90^{\circ}$  turns are performed end to end to form an S.

f. SWERVE: Variation of type e where turns are less than  $90^{\circ}$ .

g. SERPENTINE: A series of constant radius, constant angle circular arcs which oscillates the target about a mean line of travel.

h. RANDOM SERPENTINE: Similar to type g but allowing variation of turn radius and arc angle.

i. ZIG ZAG: Straight line segments connected by constant radius turns.

A-3. SYMBOLS.

#### Definitions

$V_T$	Speed of Target, M/Sec
$R_T$	Range to Target at Missile Launch, Km
$r_T$	Target Radius of Turn, M
M	Meters
Km	Kilometers

### Definitions

$t_o$	Seconds of Missile Flight prior to Target starting the Maneuver
$t_t$	Seconds of Target Maneuver prior to Missile Launch

#### A-4. TERMS.

Aspect Angle = Angle at which the gunner views the course.

Arc Angle = Degrees of arc traversed by the target in a single maneuver segment.



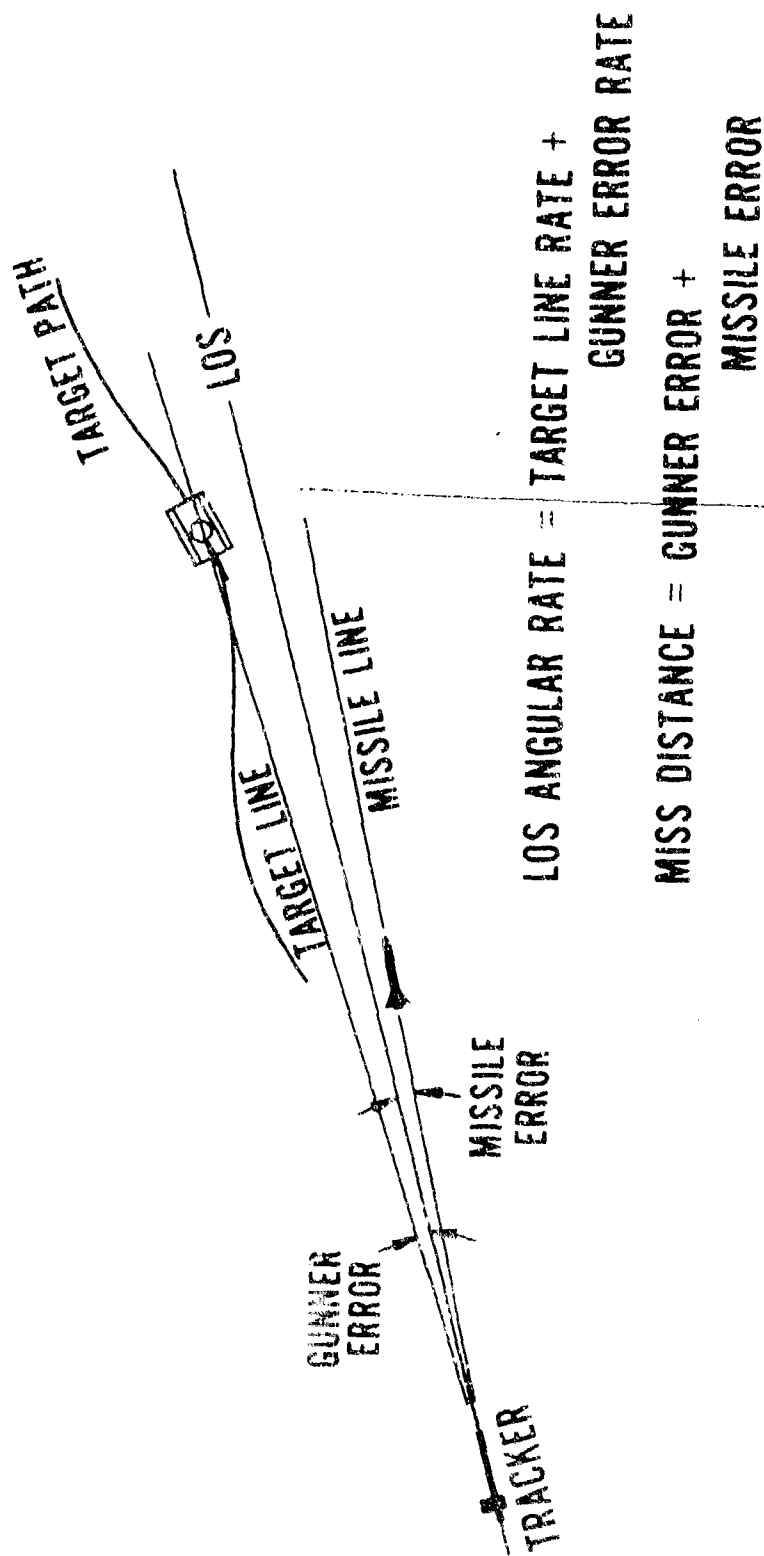


FIGURE A-1. AZIMUTH PLANE GEOMETRY

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Table A-1. Maneuvers Investigated, DRAGON

Type	No.	Target/Course Parameters	Course Geometry Figure No.	Remarks
START	1	0 to 30 mph (M551 Acceleration Curve)	None	Ranges: 1 km, 0.5 km, 0.25 km; Aspect: 90°, 60°, 30°
STOP	2	30 to 0 mph in 0 sec	None	Same as START
START/STOP	3	30 to 0 to 30 mph (Combination of 1 and 2)	None	Same as START
RIGHT TURN	4	$V_T = 6.3 \pi/\text{sec}$ ; $r_T = 20M$	Fig A-2	Ranges: 1 km, 0.5 km, 0.25 km; Aspect: 0°, to: 0 to 10 sec
LEFT TURN	5	$V_T = 13.4 \pi/\text{sec}$ ; $r_T = 30M$	Fig A-4	Ranges: 1 km, 0.5 km, 0.25 km; Aspect: 0°, 30°, 60°, 90°; $t_0$ : 0 to 10 sec
STRAIGHT	6a	$V_T = 6.94 \pi/\text{sec}$ ; $r_T = 20M$ ; Arc = 90°	Fig A-5	Ranges: 1 km, 0.5 km, 0.25 km; Aspect: 0°, 30°, 45°, 60°, 90°; $t_0$ : 0 to 10 sec
	6b	$V_T = 6.94 \pi/\text{sec}$ ; $r_T = 20M$ ; Arc = 60°		
	6c	$V_T = 6.94 \pi/\text{sec}$ ; $r_T = 20M$ ; Arc = 45°		
	6d	$V_T = 6.94 \pi/\text{sec}$ ; $r_T = 20M$ ; Arc = 30°		
DECELERATION	7a	$V_T = 6.94 \pi/\text{sec}$ ; $r_T = 20M$ ; Arc = 120°; Freq = 0.1 hz	Fig A-6	Ranges: 1 km, 0.5 km, 0.25 km; Aspect: 0°, 30°, 45°, 60°, 90°; $t_0$ : 0 to 10 sec
	7b	$V_T = 10.0 \pi/\text{sec}$ ; $r_T = 20M$ ; Arc = 90°; Freq = 0.15 hz	Fig A-7	
ACCELERATION	8a	$V_T = 6.94 \pi/\text{sec}$ ; $r_T = 20M$ (Const); Arc = Varying	Fig A-8	Ranges: 1 km, 0.5 km, 0.25 km; Aspect: 0°, 30°, 45°, 60°, 90°; $t_0$ : 0 to 10 sec
	8b	$V_T = 6.94 \pi/\text{sec}$ ; $r_T = 20, 35, 30M$ ; Arc = Varying	Fig A-9	
	8c	$V_T = 6.94 \pi/\text{sec}$ ; $r_T = 20, 35, 30M$ ; Arc = Varying	Fig A-10	
	8d	$V_T = 6.94 \pi/\text{sec}$ ; $r_T = 20, 35, 30M$ ; Arc = Varying	Fig A-11	
STOP/START	9	$V_T = 6.94 \pi/\text{sec}$ ; $r_T = 20M$ ; Arc = 90° + St. Line	Fig A-12	Ranges: 1 km, 0.5 km, 0.25 km; Aspect: 0°, 30°, 45°, 60°, 90°; $t_0$ : 0 to 10 sec

Table A-2. Maneuvers Investigated, TOW

Maneuver	Target/Course Parameters	Course Geometry Figure No.	Remarks
START	0 to 30 m/s (M557 Acceleration Curve)	None	Ranges: 3 km, 2 km, 1 km, Aspect: 90°
STOP	30 to 0 m/s in 5 sec	None	Same as START
STATIONARY	30 to 0 to 30 m/s (Combination of 1 and 2)	None	Range: 3 km; Aspect: 90°
PILOT TURN	$V_T = 8.34 \text{ m/sec}$ ; $r_T = 20\text{M}$	Fig A-2	Ranges: 3 km, 2 km, 1 km; Aspect: 0°, $t_0$ : 0 to 14 sec
C TURN	$V_T = 13.4 \text{ m/sec}$ ; $r_T = 30\text{M}$	Fig A-4	Ranges: 3 km, 2 km, 1 km; Aspect: 0°, 30°, 60°, 90°, $t_0$ : 0 to 14 sec
SWITCH	$V_T = 8.34 \text{ m/sec}$ ; $r_T = 20\text{M}$ ; Arc = 90° $V_T = 8.34 \text{ m/sec}$ ; $r_T = 20\text{M}$ ; Arc = 60° $V_T = 8.34 \text{ m/sec}$ ; $r_T = 20\text{M}$ ; Arc = 45° $V_T = 8.34 \text{ m/sec}$ ; $r_T = 20\text{M}$ ; Arc = 30°	Fig A-5	Ranges: 3 km, 2 km, 1 km; Aspect: 0°, 30°, 45°, 60°, 90°; $t_0$ : 0 to 14 sec
EXPLOSION	$V_T = 8.34 \text{ m/sec}$ ; $r_T = 20\text{M}$ ; Arc = 120°; Freq = 0.1 hz $V_T = 10.0 \text{ m/sec}$ ; $r_T = 20\text{M}$ ; Arc = 30°; Freq = 0.15 hz	Fig A-6 Fig A-7	Ranges: 3 km, 2 km, 1 km; Aspect: 0°, 30°, 45°, 60°, 90°; $t_0$ : 0 to 14 sec
EXPLOSION	$V_T = 8.34 \text{ m/sec}$ ; $r_T = 20\text{M}$ (Const); Arc = Varying $V_T = 8.34 \text{ m/sec}$ ; $r_T = 20, 25, 30\text{M}$ ; Arc = Varying $V_T = 8.34 \text{ m/sec}$ ; $r_T = 20, 25, 30\text{M}$ ; Arc = Varying $V_T = 8.34 \text{ m/sec}$ ; $r_T = 20, 25, 30\text{M}$ ; Arc = Varying	Fig A-8 Fig A-9 Fig A-10 Fig A-11	Ranges: 3 km, 2 km, 1 km; Aspect: 0°, 30°, 45°, 60°, 90°; $t_0$ : 0 to 12 sec
STOP TURN	$V_T = 8.34 \text{ m/sec}$ ; $r_T = 20\text{M}$ ; Arc = 90° + St. Line	Fig A-12	Ranges: 3 km, 2 km, 1 km; Aspect: 0°, 30°, 45°, 60°, 90°; $t_0$ : 0 to 12 sec

Table A-6. Maneuvers Investigated, Shillelagh

Maneuver	Target/Source Parameters	Course Geometry Figure No.	Remarks
1. Acceleration	Acceleration = 500, 30, 20, 15, 10 mph to 0, acceleration to 0, 100	None	Ranges: 3 km, 2 km, 1 km; Aspect: 90°
2. Acceleration	Acceleration = 500, 30, 20, 15, 10 mph to 0, acceleration to 0, 100	None	Same as START
3. Acceleration	Acceleration = 500, 30, 20, 15, 10 mph to 0, acceleration to 0, 100	None	Ranges: 3 km, 2 km, 1 km; Aspect: 90°; Hesitation: 0 to 5 sec
4. Acceleration	Acceleration = 500, 30, 20, 15, 10 mph to 0, acceleration to 0, 100	Fig A-2	Ranges: 3 km, 2 km, 1 km; Aspect: 90°; $t_0$ (4): 0 to 14 sec; $t_0$ (4a): 0 to 5.6 sec
5. Acceleration	Acceleration = 500, 30, 20, 15, 10 mph to 0, acceleration to 0, 100	Fig A-3	Ranges: 3 km, 2 km, 1 km; Aspect: 90°; $t_0$ (4): 0 to 14 sec; $t_0$ (4a): 0 to 5.6 sec
6. Acceleration	Acceleration = 500, 30, 20, 15, 10 mph to 0, acceleration to 0, 100	Fig A-5	Ranges: 3 km, 2 km, 1 km; Aspect: 90°; $t_0$ (4): 0 to 14 sec; $t_0$ (4a): 0 to 5.6 sec
7. Acceleration	Acceleration = 500, 30, 20, 15, 10 mph to 0, acceleration to 0, 100	Fig A-6	Ranges: 3 km, 2 km, 1 km; Aspect: 90°; $t_0$ (4): 0 to 14 sec; $t_0$ (4a): 0 to 5.6 sec
8. Acceleration	Acceleration = 500, 30, 20, 15, 10 mph to 0, acceleration to 0, 100	Fig A-7	Ranges: 3 km, 2 km, 1 km; Aspect: 90°; $t_0$ (4): 0 to 14 sec; $t_0$ (4a): 0 to 5.6 sec
9. Acceleration	Acceleration = 500, 30, 20, 15, 10 mph to 0, acceleration to 0, 100	Fig A-8	Ranges: 3 km, 2 km, 1 km; Aspect: 90°; $t_0$ (4): 0 to 14 sec; $t_0$ (4a): 0 to 5.6 sec
10. Acceleration	Acceleration = 500, 30, 20, 15, 10 mph to 0, acceleration to 0, 100	Fig A-9	Ranges: 3 km, 2 km, 1 km; Aspect: 90°; $t_0$ (4): 0 to 14 sec; $t_0$ (4a): 0 to 5.6 sec
11. Acceleration	Acceleration = 500, 30, 20, 15, 10 mph to 0, acceleration to 0, 100	Fig A-10	Ranges: 3 km, 2 km, 1 km; Aspect: 90°; $t_0$ (4): 0 to 14 sec; $t_0$ (4a): 0 to 5.6 sec
12. Acceleration	Acceleration = 500, 30, 20, 15, 10 mph to 0, acceleration to 0, 100	Fig A-11	Ranges: 3 km, 2 km, 1 km; Aspect: 90°; $t_0$ (4): 0 to 14 sec; $t_0$ (4a): 0 to 5.6 sec
13. Acceleration	Acceleration = 500, 30, 20, 15, 10 mph to 0, acceleration to 0, 100	Fig A-12	Ranges: 3 km, 2 km, 1 km; Aspect: 90°; $t_0$ (4): 0 to 14 sec; $t_0$ (4a): 0 to 5.6 sec

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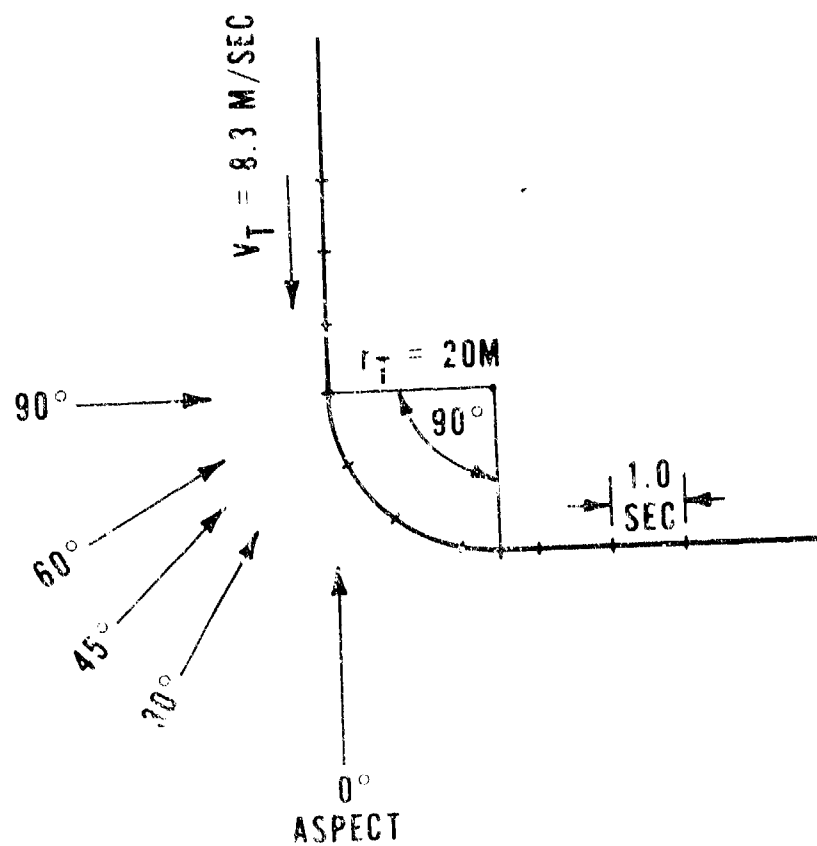


FIGURE A-2. FAST TURN, COURSE 4.

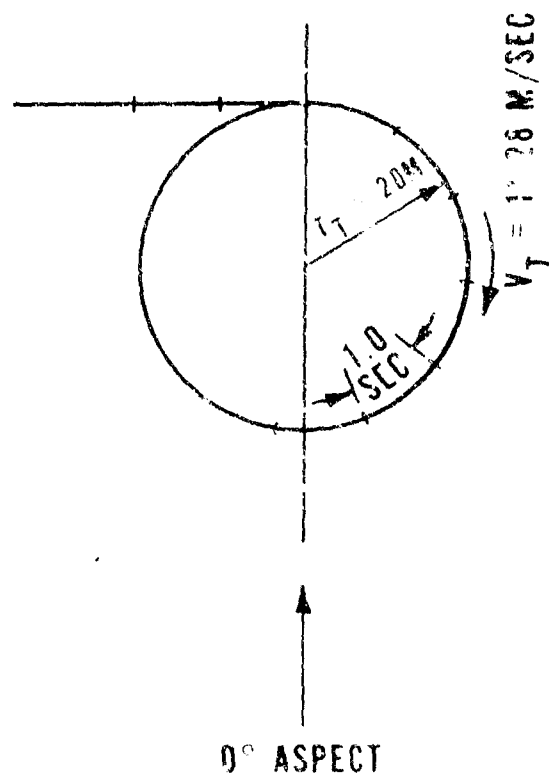


FIGURE A-3. FAST TURN (CIRCULAR),  
COURSE 4a.

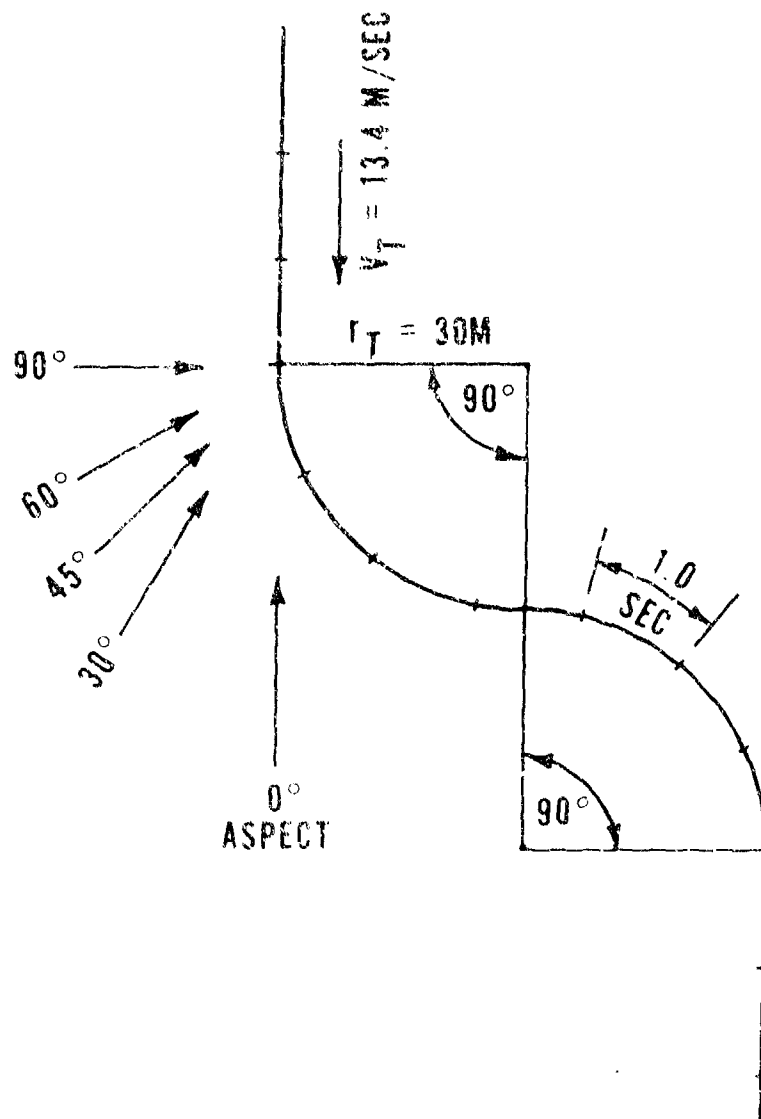


FIGURE A-4. "S" TURN, COURSE 5.

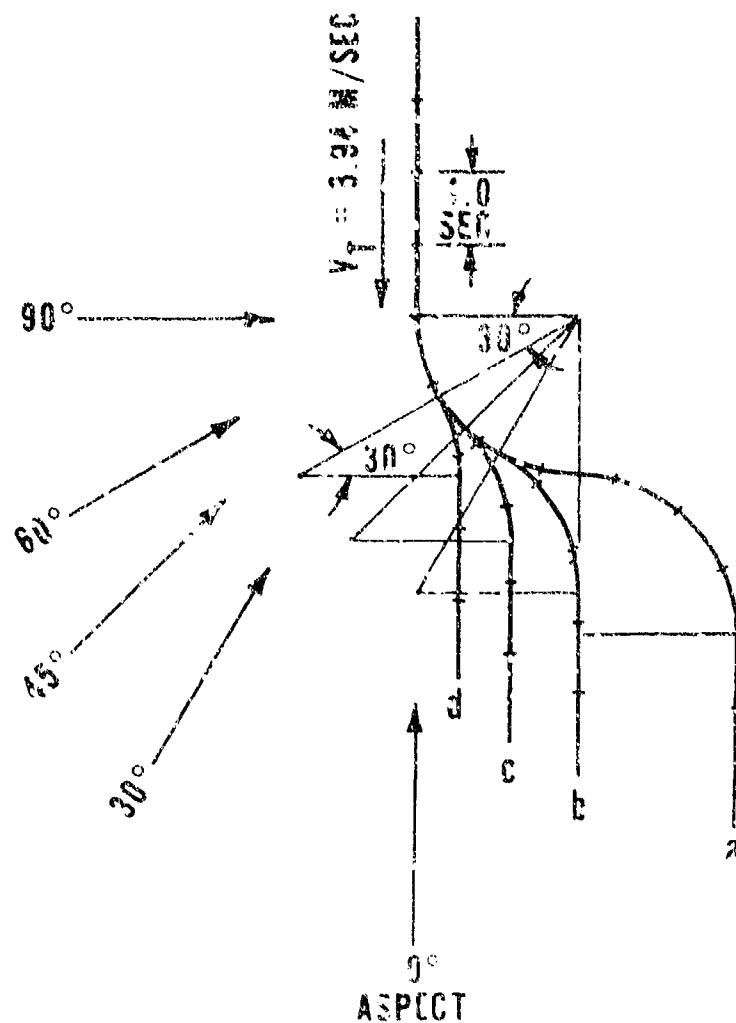


FIGURE A-5. SWERVE, COURSES 6a,b,c,& d.



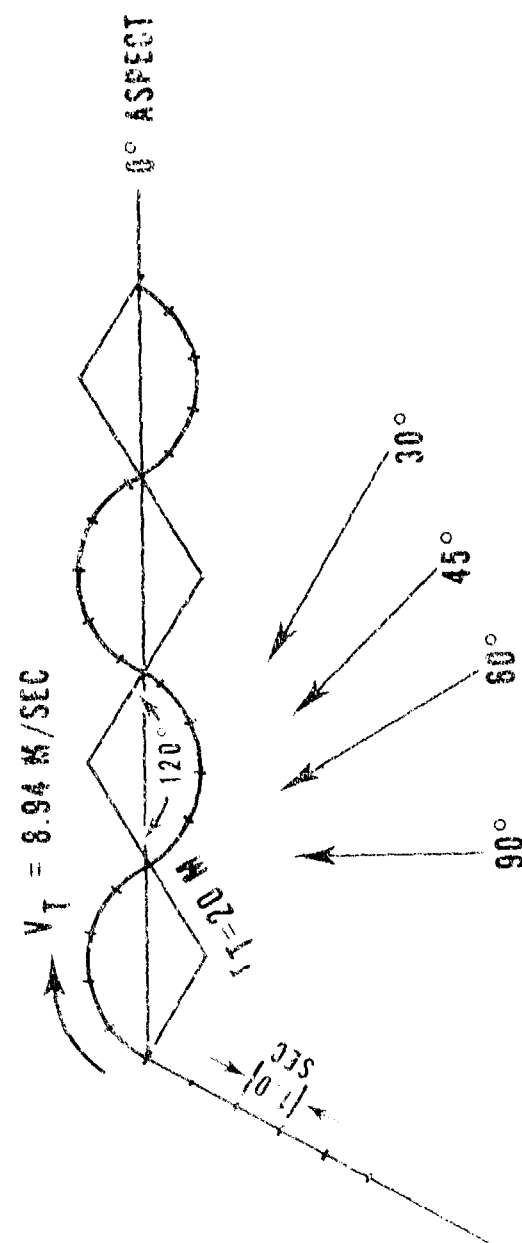


FIGURE A-6.0.1 HZ SERPENTINE, COURSE 7a.

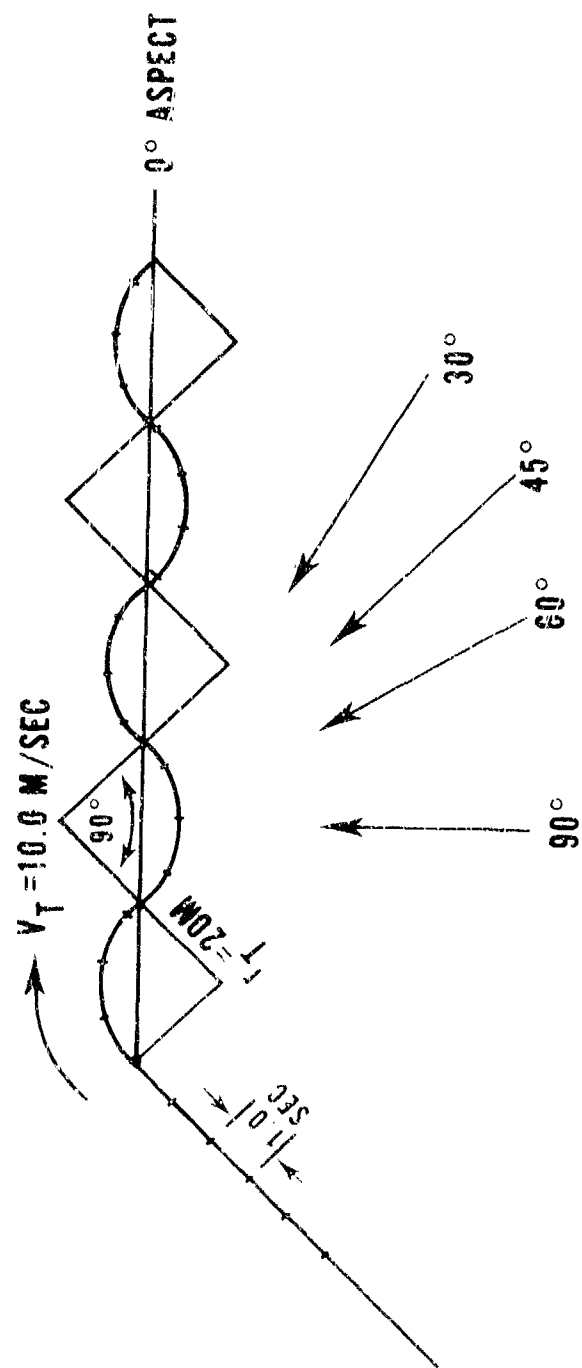


FIGURE A-7. 0.15 HZ SERPENTINE, COURSE 7b.

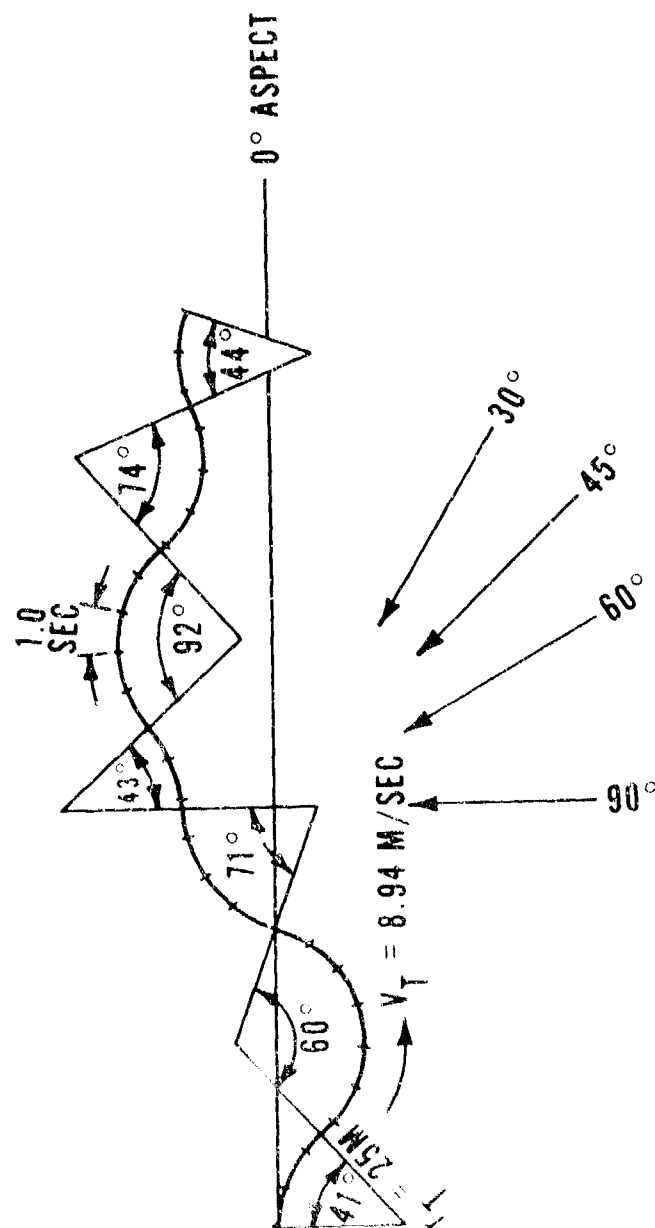


FIGURE A-3. RANDOM SERPENTINE, COURSE 8a.

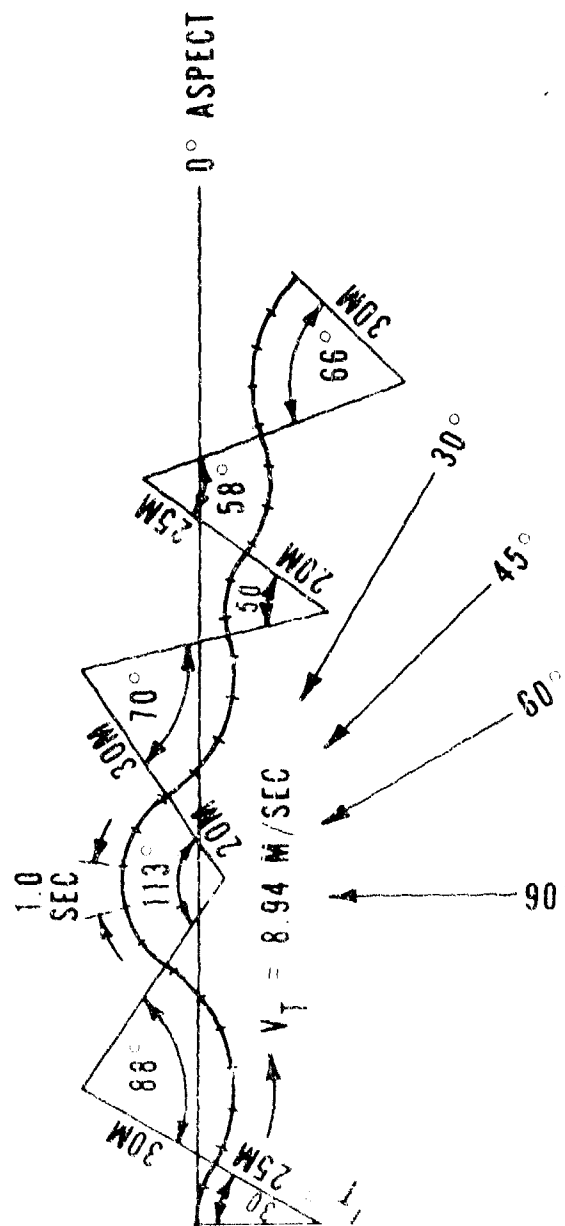


FIGURE A-9. RANDOM SERPENTINE, COURSE 8b.

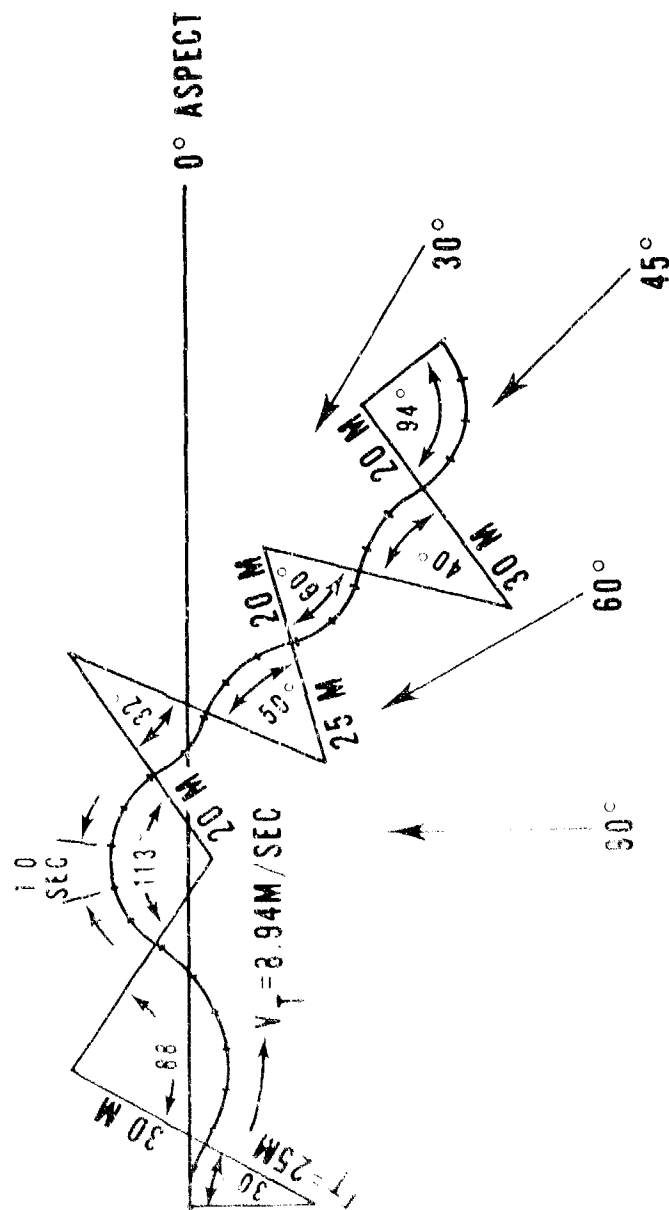


FIGURE A-10. RANDOM SERPENTINE, COURSE 8C.

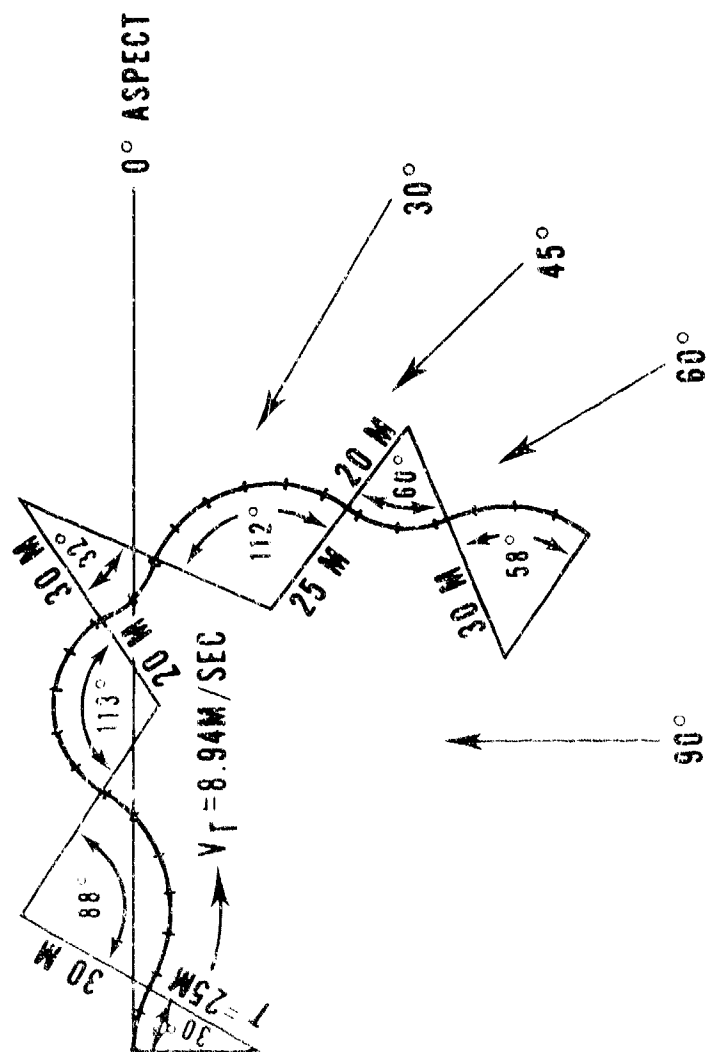


FIGURE A-11. RANDOM SERPENTINE, COURSE 8d.

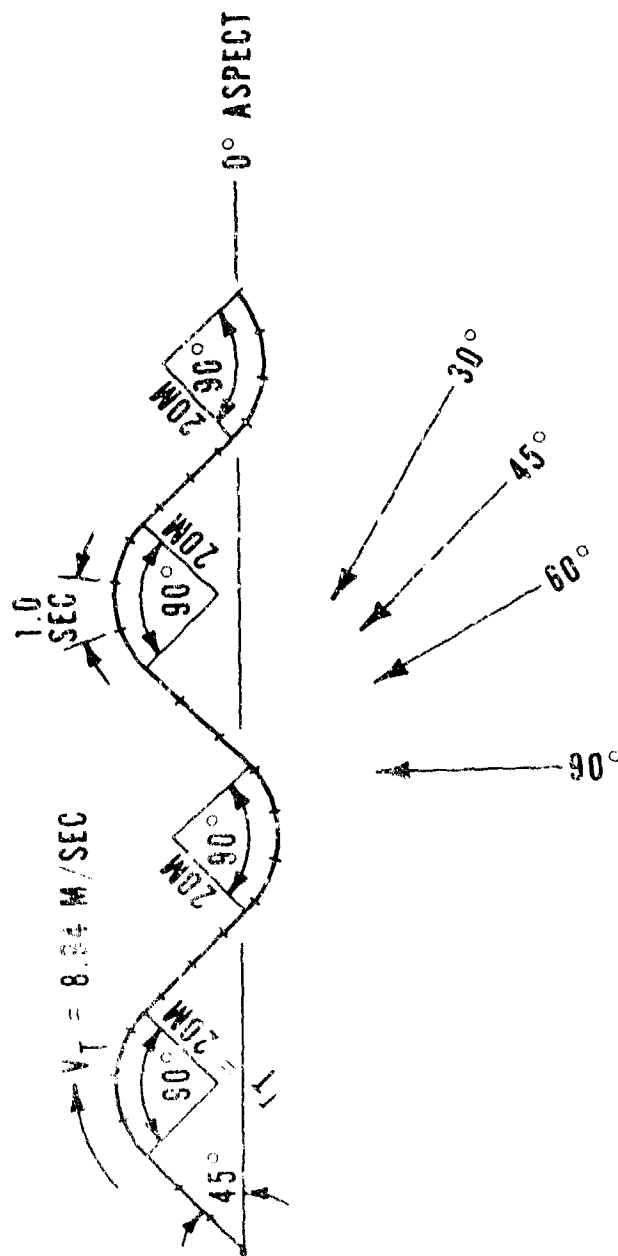


FIGURE A-12. ZIG-ZAG, COURSE 9.

## APPENDIX B

### INTERVISIBILITY COMBINATIONS FOR MISSILE ENGAGEMENT SEQUENCES

B-1. TABLES B-1 THROUGH B-3. The number and percentages of LOS fire and impact intervisibility combinations occurring across all ranges are shown in these tables. Each table identifies the combinations, categorized by weapon system, against each specific target vehicle.

B-2. TABLES B-4 AND B-5. These tables depict the number and percentages of LOS fire and impact intervisibility combinations against all target vehicles. Each table identifies the combinations, categorized by weapon system, in each of the two range bands.

B-3. TABLES B-6 THROUGH B-11. The intervisibility combinations in these tables have been identified by weapon system, but each table has been arrayed to represent only one target vehicle in only one range band.



Table B-1. LOS fire and impact intervisibility combinations against the M60A1 tank target vehicle

Condition	Weapon System							
	DASH				TOW			
	Number	Percent	Number	Percent	Number	Percent	Number	Percent
LOS-LOS	77	67.54	433	86.08	148	82.22	188	87.04
LOS-LO	9	7.90	23	4.57	7	3.89	7	3.24
LOS-LOS	6	5.26	14	2.78	10	5.56	9	4.17
LOS-LOS	5	4.39	6	1.19	5	2.78	2	.92
LOS-LO	17	14.91	26	5.18	9	5.00	9	4.17
LOS-LOS	0	0.00	1	.20	1	.55	1	.46
TOTAL	114	100.00	503	100.00	180	100.00	216	100.00

Table E-1. 100 fire and impact intervisibility combinations against the XM800 Scout target vehicle

Weapon	Weapon System					
	M501		M501		M501/2	
	Number	Percent	Number	Percent	Number	Percent
100-100	60	86.96	289	83.77	90	72.58
100-75	2	2.90	12	3.48	11	8.87
100-50	1	1.45	29	8.40	4	3.23
100-25	0	0.00	4	1.16	7	5.64
75-100	6	8.69	11	3.19	8	6.45
75-75	0	0.00	0	0.00	4	3.23
75-50	0	0.00	0	0.00	0	0.00
75-25	0	0.00	0	0.00	0	0.00
TOTAL	69	100.00	345	100.00	124	100.00

Table 5-3. LOS fire and impact intervisibility combinations against the XM808 TWISTER target vehicle

LOS Condition	Weapon System											
	DRAGON				TOW				M551			
	Number	Percent	Number	Percent	Number	Percent	Number	Percent	Number	Percent	Number	Percent
LOS-LOS	20	66.67	133	76.44	37	88.10	37	92.50				
LOS-IR	5	16.66	7	4.02	3	7.14	0	0.00				
LOS-LOS	2	6.67	2	1.15	1	2.38	2	5.00				
IR-LOS	3	10.00	5	2.87	1	2.38	1	2.50				
IR-IR	0	0.00	23	13.22	0	0.00	0	0.00				
IR-LOS	0	0.00	4	2.30	0	0.00	0	0.00				
TOTAL	30	100.00	174	100.00	42	100.00	40	100.00				

Table B-4. LOS fire and impact intervisibility combinations in range bands 1 and 2 against all target vehicles

LOS Condition	Weapon System									
	DRAGON		TOW		M551		M60A2			
	Number	Percent	Number	Percent	Number	Percent	Number	Percent		
LOS-LOS	157	73.71	464	89.58	109	76.76	126	84.00		
LOS-LOS	16	7.51	9	1.74	10	7.04	5	3.33		
LOS-NLOS	9	4.22	4	.77	0	0.00	2	1.33		
LOS-LOS	8	3.76	7	1.35	7	4.93	3	2.00		
LOS-LOS	23	10.80	34	6.56	16	11.27	14	9.34		
LOS-NLOS	0	0.00	0	0.00	0	0.00	0	0.00		
TOTAL	213	100.00	518	100.00	142	100.00	150	100.00		

Table B-5. LOS fire and impact intervisibility combinations in range bands 3 and 4 against all target vehicles

LOS Condition	Weapon System					
	TOW		M551		M60A2	
	Number	Percent	Number	Percent	Number	Percent
LOS-LOS	391	77.58	166	81.37	188	87.95
LOS-TV	33	6.55	11	5.39	10	4.67
LOS-A200	41	8.13	15	7.36	14	6.54
TV-LOS	8	1.59	6	2.94	1	.47
TV-TV	26	5.16	1	.49	0	0.00
TV-A200	5	.99	5	2.45	1	.47
TOTAL	504	100.00	204	100.00	214	100.00

Table B-6. LOS fire and impact intervisibility combinations in range bands 1 and 2 against the M60A1 tank target vehicle

Condition	Weapon System									
	DRAGON				TOW			M551		
	Number	Percent	Number	Percent	Number	Percent	Number	Percent	Number	Percent
LOS-LOS	77	67.54	256	89.51	57	75.00	78	82.10		
LOS-TV	9	7.90	5	1.75	6	7.90	4	4.21		
LOS-ALOS	6	5.26	2	.70	0	0.00	2	2.11		
TV-ALOS	5	4.39	3	1.05	4	5.26	2	2.11		
TV-TV	17	14.91	20	6.99	9	11.84	9	9.47		
TV-ALOS	0	0.00	0	0.00	0	0.00	0	0.00		
TOTAL	114	100.00	286	100.00	76	100.00	95	100.00		

Table B-7. LOS fire and impact intervisibility combinations in range bands 1 and 2 against the XM800 Scout target vehicle

Condition	Weapon System							
	BGM-71		TOW		M551		M60A2	
	Number	Percent	Number	Percent	Number	Percent	Number	Percent
LOS-LOS	60	86.96	130	90.91	27	69.23	27	81.82
LOS-LOS	2	2.90	1	.70	2	5.13	1	3.03
LOS-LOS	1	1.45	2	1.40	0	0.00	0	0.00
LOS-LOS	0	0.00	0	0.00	3	7.69	0	0.00
LOS-LOS	6	8.69	10	6.99	7	17.95	5	15.15
LOS-LOS	0	0.00	0	0.00	0	0.00	0	0.00
TOTAL	69	100.00	143	100.00	39	100.00	33	100.00

Table B-3. LOS fire and impact intervisibility combinations in range bands 1 and 2 against the XM808 TWISTER target vehicle

Condition	Weapon System							
	DRAGON		TOW		M551		M60A2	
	Number	Percent	Number	Percent	Number	Percent	Number	Percent
LOS-LOS	20	66.67	78	87.64	25	92.59	21	95.45
LOS-IV	5	16.67	3	3.38	2	7.41	0	0.00
LOS-VLOS	2	6.66	0	0.00	0	0.00	0	0.00
IV-LOS	3	10.00	4	4.49	0	0.00	1	4.55
IV-IV	0	0.00	4	4.49	0	0.00	0	0.00
IV-VLOS	0	0.00	0	0.00	0	0.00	0	0.00
TOTAL	30	100.00	89	100.00	27	100.00	22	100.00



Table B-9. LOS fire and impact intervisibility combinations in range bands 3 and 4 against the M60A1 tank target vehicle

LOS	Weapon System					
	M551			M60A2		
LOS	Number	Percent	Number	Percent	Number	Percent
LOS 1	177	81.57	91	87.50	110	90.91
LOS 2	18	8.29	1	.96	3	2.48
LOS 3	12	5.53	10	9.62	7	5.78
LOS 4	3	1.38	1	.96	0	0.00
LOS 5	6	2.76	0	0.00	0	0.00
LOS 6	1	.47	1	.96	1	.83
TOTAL	217	100.00	104	100.00	121	100.00

Table B-10. LOS fire and impact intervisibility combinations in range bands 3 and 4 against the XM800 Scout target vehicle

LOS Condition	Weapon System					
	TOS		M551		M60A2	
	Number	Percent	Number	Percent	Number	Percent
LOS-LOS	159	78.71	63	74.12	62	82.67
LOS-IV	11	5.44	5	10.50	7	9.33
LOS-ALOS	27	13.37	4	4.70	5	6.67
IV-LOS	4	1.98	4	4.70	1	1.33
IV-IV	1	.50	1	1.18	0	0.00
IV-ALOS	0	0.00	4	4.70	0	0.00
TOTAL	202	100.00	85	100.00	75	100.00

Table B-11. LOS fire and impact intervisibility combinations in range bands 3 and 4 against the XM808 TWISTER target vehicle

LOS Band	Aedon System					
	M551			M60A2		
	Count	Value	Percent	Count	Value	Percent
Band 3	35	64.70	12	16	88.89	88.89
Band 4	4	4.71	1	0	0.00	0.00
Total	39	69.41	13	16	88.89	88.89
Band 3	19	22.35	0	0	0.00	0.00
Band 4	4	4.71	0	0	0.00	0.00
Total	23	27.06	0	0	0.00	0.00
Total	85	100.00	15	18	100.00	100.00

## APPENDIX C

### PERCENTAGES OF TIME IN LOS CONDITIONS

C-1. TABLES C-1 THROUGH C-3. The percentage of total trial time the target vehicles spent in each LOS condition for all ranges is portrayed in these tables. Each table is arrayed with a single target vehicle against each weapon system.

C-2. TABLES C-4 AND C-5. The combinations in these tables identify the LOS condition by range band and weapon system against all target vehicles.

C-3. TABLES C-6 THROUGH C-11. These tables depict the LOS conditions categorized by weapon system. Each table presents those conditions for a single target vehicle and a single range band.

Table C-1. The percentage of total trial time spent by the M60A1 tank in each LOS condition, as observed by each weapon system.

LOS Condition	Weapon System			
	DRAGON	HOW	M551	M60A2
LOS	68.96	66.29	56.22	55.98
IV	17.23	17.38	18.16	14.55
NILOS	14.81	16.33	25.62	29.47
TOTAL	100.00	100.00	100.00	100.00

Table C-2. The percentage of total trial time spent by the XM800 Scout in each LOS condition, as observed by each weapon system

LOS Condition	Weapon System			
	DRAGON	10W	M551	M60A2
I OS	71.26	68.78	52.77	55.57
IV	12.87	11.29	24.53	23.09
RI OS	15.87	19.93	22.70	21.34
101A2	100.00	100.00	100.00	100.00

Table C-3. The percentage of total trial time spent by the XM808 TWISTER in each LOS condition, as observed by each computer system

LOS Condition	Computer System			
	IBM 360	IBM	IBM 370	IBM 370/2
IOS	67.75	67.75	67.32	62.00
IV	19.68	19.68	14.81	15.75
MEOS	12.57	12.57	17.87	22.25
TOTAL	100.00	100.00	100.00	100.00

Table C-4. The percentage of trial time spent in each LOS condition for all target vehicles in range bands 1 and 2, as observed by each weapon system

LOS Condition	Weapon System			
	ERAGON	LOW	M551	M60A2
I OS	69.30	69.95	51.23	51.08
IV	16.41	15.89	24.14	24.93
RE OS	14.29	14.16	24.63	23.99
TOTAL	100.00	100.00	100.00	100.00



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Table C-5. The percentage of trial time spent in each LOS condition for all target vehicles in range bands 3 and 4, as observed by each weapon system

LOS Condition	Weapon System		
	TOW	M551	M60A2
LOS	63.20	58.72	60.13
IV	16.63	16.63	19.07
NLOS	20.17	24.65	20.86
TOTAL	100.00	100.00	100.00

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Table C-6. The percentage of trial time spent in each LOS condition by the M60A1 tank in range bands 1 and 2

LOS Condition	Weapon System			
	DRAGON	TOW	M551	M60A2
I OS	68.86	69.69	53.03	52.57
IV	17.23	16.90	22.89	23.72
III OS	13.91	13.41	24.08	23.71
TOTAL	100.00	100.00	100.00	100.00

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Table C-7. The percentage of trial time spent in each LOS condition by the XM800 Scout in range bands 1 and 2

LOS Condition	Weapon System			
	DRAGON	TOW	M551	M60A2
LOS	71.26	71.26	38.99	38.99
IV	12.87	12.87	32.59	33.93
NLOS	15.87	15.87	28.42	27.08
TOTAL	100.00	100.00	100.00	100.00

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Table C-8. The percentage of trial time spent in each LOS condition by the XM808 TWISTER in range bands 1 and 2

LOS Condition	Weapon System			
	DRAGON	TOW	M551	M60A2
I OS	67.52	68.84	63.41	63.41
IV	19.58	16.71	15.77	16.72
III OS	12.90	14.45	20.82	19.87
TOTAL	100.00	100.00	100.00	100.00

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Table C-9. The percentage of trial time spent in each LOS condition by the M60A1 tank in range bands 3 and 4

LOS Condition	Weapon System		
	TOW	M551	M60A2
LOS	63.45	58.60	58.94
IV	17.79	14.62	19.67
III OS	18.76	26.78	21.39
TOTAL	100.00	100.00	100.00

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Table C-10. The percentage of trial time spent in each LOS condition by the XM800 Scout in range bands 3 and 4

LOS Condition	Weapon System		
	10W	M551	M60A2
LOS	67.66	58.49	62.45
IV	10.58	21.19	18.59
III OS	21.76	20.32	18.96
TOTAL	100.00	100.00	100.00

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Table C-11. The percentage of trial time spent in each LOS condition by the XM808 TWISTER in range bands 3 and 4

LOS Condition	Weapon System		
	TOW	M551	M60A2
LOS	49.47	61.18	60.53
IV	29.49	13.82	14.14
NLOS	21.04	25.00	25.33
TOTAL	100.00	100.00	100.00

## APPENDIX D

### MATHEMATICAL DERIVATION OF CUBIC LEAST SQUARES FIT POLYNOMIAL

#### D-1. STATEMENT OF THE PROBLEM.

a. The Range Measuring System (RMS) provided position data for the ATMT target vehicles in terms of time, UTM x-coordinates, and UTM y-coordinates. The z-coordinate values were added by CDEC based on 10-meter grid aerial survey data and a cubic spline interpolation technique (reference 1). In order to characterize target vehicle movement, it was necessary to obtain the velocity and acceleration in the UTM coordinate system as a function of time.

b. A direct computation of differences in the position coordinates divided by differences in time showed the data contained far too many errors to give realistic estimates of velocity and acceleration. Several different methods were tried to smooth the coordinates and obtain realistic velocity and acceleration as functions of time (reference 2). The methods considered included point skipping methods, spline curve fitting, Kalman filtering, and least squares curve fitting. The method selected was the fitting of cubic polynomials to  $N \geq 4$  time-coordinate observed data points. The velocities and accelerations were computed from the first and second derivatives of the positions expressed as cubic polynomials of the form:

$$\begin{aligned}x(t) &= a_0 + a_1t + a_2t^2 + a_3t^3 \\y(t) &= b_0 + b_1t + b_2t^2 + b_3t^3 \\z(t) &= c_0 + c_1t + c_2t^2 + c_3t^3\end{aligned}\tag{D-1}$$

The coefficients were actually determined from a least squares fit to 35 (an odd number for symmetry purposes) observed data points.

c. The derivation of  $x(t)$  is presented in paragraph D-2, with the derivations of the other coordinates being similar. This derivation is presented since most books and packaged programs employ equally spaced (in time or the independent variable) data points. The luxury of equally spaced points is not present in the RMS data. Most, but not all, data points occurred each  $.1 \pm .01$  seconds. For less than 9 observed data points, the accelerations obtained were extremely unrealistic; e.g., they had wide variations and jumped from positive to negative to positive in one-half second. An autocorrelation program on some of the data suggested that at least 20 points were needed before the errors (plus some true signal) became independent of each other. This is in agreement with the heuristic reasoning that the vehicle could make at least one but not more than one major acceleration change each one to two seconds. This is the same number of second derivative sign changes possible on each axis with a



cubic polynomial. The remaining consideration is determining the number of data points to use in order to obtain both smooth and realistic velocities and accelerations. Based upon Calcomp plots of the resultant velocities and accelerations as well as their components, a 35 point set was finally chosen for the estimation process.

## D-2. DERIVATION OF THE COEFFICIENTS.

a. Analytics. The notation used in this paragraph is  $I$  for the number of points fitted,  $x(t)$  for the actual x-coordinate (unknown),  $\bar{x}(t)$  for the observed x-coordinate,  $\hat{x}(t)$  for the smoothed x-coordinate, and  $t_k$  for the time in question (where  $(I+1)/2 \leq k \leq I-(I-1)/2$ ). With this notation the smoothed x-coordinate is:

$$\hat{x}(t_k) = a_0 + a_1 t_k + a_2 t_k^2 + a_3 t_k^3,$$

the  $\hat{x}$  component of velocity at  $t_k$  is:

$$\frac{d\hat{x}(t_k)}{dt_k} = a_1 + 2a_2 t_k + 3a_3 t_k^2,$$

and the  $\hat{x}$  component of acceleration at  $t_k$  is:

$$\frac{d^2 \hat{x}(t_k)}{dt_k^2} = 2a_2 + 6a_3 t_k.$$

Let  $M = (I-1)/2$ . The objective in least squares fit is to minimize:

$$\begin{aligned} H(a_0, a_1, a_2, a_3) &= \sum_{i=k-M}^{k+M} [\bar{x}(t_i) - x(t_i)]^2 \\ &= \sum_{i=k-M}^{k+M} \left[ x(t_i) - \sum_{j=0}^3 a_j (t_i)^j \right]^2 \end{aligned} \quad (D-2)$$

To find the coefficients by minimizing  $H$ , the partial derivative of  $H$  with respect to each  $a_n$  must be calculated, the resulting expressions set equal to zero, and the  $a_n$ 's solved for each  $n = 0, 1, 2, 3$ . Thus, (for each  $n$ ):

$$\frac{\partial H}{\partial a_n} = \sum_{i=k-M}^{k+M} 2(\bar{x}(t_i) - \sum_{j=0}^3 a_j (t_i)^j) (-t_i)^n = 0; \quad (D-3)$$

and by simplifying (including switching the order of addition):

$$\sum_{j=0}^3 \left( \sum_{i=k-M}^{k+M} (t_i)^{j+n} \right) a_j = \sum_{i=k-M}^{k+M} \bar{x}(t_i) (t_i)^n. \quad (D-4)$$

In matrix form this last system of equations is  $BA=R$  or  $A=B^{-1}R$  where:

$$A = (a_n); \quad r = (r_n); \quad b = (b_{n,j});$$

$$r_n = \sum_{i=k-M}^{k+M} \bar{x}(t_i) (t_i)^n, \quad n = 0, 1, 2, 3; \quad (D-5)$$

and

$$b_{nj} = \sum_{i=k-M}^{k+M} (t_i)^{n+j} \quad n, j = 0, 1, 2, 3.$$

b. Cautions. The equations of the previous paragraph must be applied with care when programmed on a computer. If the size of  $t_i$  is large relative to  $t_i - t_{i-1}$  (e.g., the number of significant digits in  $t_i$  minus the number of significant digits in  $t_i - t_{i-1}$  is greater than machine precision), then it is required to translate the  $t_i$ 's so that  $t_k = 0$  in order to achieve any numerical accuracy. After a translation it can be observed that if the  $t_i$  were equally spaced, then  $B$  (already a symmetric matrix) is a constant matrix, a fact which could be used to shorten the number of computations. If an orthogonal set of polynomials could be used (only reasonably possible in the case of equal spacing) the matrix  $B$  would be diagonal and easy to invert.

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1. US Army Combat Developments Experimentation Command (1976), Antitank Missile Test (ATMT) Final Report, Phase II, Annex D, USACDEC, Fort Ord, California
2. US Army Combined Arms Combat Developments Activity (1977), Combat Operations Analysis Directorate, Technical Paper 11-77, Filtering and Smoothing of Time Related Data, USACACDA, Fort Leavenworth, Kansas

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